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ABSTRACT

This guide is a continuation of the series of guides starting with SE 012 723. The entire course is ungraded, but the material referred to in this guide would probably be covered in grade five or six. The concept of place value is studied more deeply using Dienes' Multiple Attribute Blocks, and used in the further development of formal algorithms. The study of fractions is continued, and decimal fractions are introduced. The commutative, associative, and distributive laws are again stressed, and used in computation and manipulation of equations. Applied number activities in length, area, volume, capacity, weight, time and money lead to greater precision in measurement and knowledge of further units, and contain a greater emphasis on calculation. The guide also contains suggestions for work on spatial relations (including nets of solids and angles) and statistics and graphs (drawing and interpretation). (MM)

CURRICULUM GUIDE
MATHEMATICS

SECTION G

EDUCATION DEPARTMENT OF VICTORIA

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CONTENTS

INTRODUCTION

Mathematics development should be seen as an " on-going " process. Children beginning a topic at Section G level should have adequately mastered the prerequisites for it in Section F. Understandings and skills gained in Section G should enable the child to handle the topic at Section H level. For example, the child in Section F has individual experiences with concrete materials leading to an informal understanding of the basic ideas of arithmetic (commutative, associative, distributive, and identity properties of number). In Section G these ideas are needed for the child to understand the algorithms of addition, multiplication, subtraction, and division. In Section H and subsequently, these algorithms are extended to compound quantities and, as skills, are used in the child's attack on problem situations.

The value of first-hand experience with concrete materials has been emphasized in previous sections. This experience should be provided for in Section G. Experiences gained in the primary school will form a background of great value to the child for his present needs and also for his future needs when he meets more formalized situations in the secondary school. This is exemplified especially in his experience with the use of materials such as Cuisenaire rods, Dienes's Multibase Arithmetic Blocks, the abacus, and materials used in spatial relations and measurement topics.

This mathematics course is an ungraded course. Not only will different children begin Section G at different times, but the individual child is most unlikely to begin all of the Section G topics at the one time. His specific abilities and interests will enable him to complete some Section F topics before others. One child may be ready to begin work in, say, volume at Section G level before he has completed his work in fractions at Section F level. Teachers should, however, beware of allowing a too extensive development in one topic to occur at the expense of other topics.

Topics are developed in the Guide as complete units. In this form it is considered that the reader can more easily gain an over-all appreciation of the development. It is not intended, however, that topics should be treated in isolation from one another in the classroom. On the contrary, it is important to integrate topics and so make the over-all structure of mathematics more meaningful to the child. A study of volume can well involve spatial relations, pattern and order in the number system, basic properties, and equations—these links are emphasized in the Guide under Volume.

Opportunity should be taken to integrate the mathematics content with other subjects of the curriculum. Statistics and graphs are given meaning in a social studies context. Measurement is especially appropriate in science.

A child's learning of mathematics is essentially an individual process ; he must do his own learning. However, the process is facilitated (considerably in some topics) by his participation in group activity. Such activity may well include class discussion led by the teacher, and also participation in more intimate, smaller groups, say, of two or three children. Work with materials in applied number particularly lends itself to group treatment.

It is important to refer to the Guides to the earlier sections, especially that for Section F. The introductions to—

Course of Study for Primary Schools, Mathematics, 1964 ;

Course of Study for Primary Schools, Mathematics (Pure and Applied Number, Sections G, H, and I), 1967 ;

Curriculum Guide, Applied Number Course, Sections A, B, C ; and *Sections D, E, F*;

Curriculum Guide, Pure Number, Section F

should also be read to assist in gaining a full appreciation of both the content and the spirit of the course.

This Guide has been prepared for the information of teachers. It is emphasized that the developments are suggestive only ; the Guide simply presents an interpretation of the course material. Teachers should be selective in its use and should feel free to adapt it to suit their own particular class situations.

PATTERN AND ORDER IN THE NUMBER SYSTEM

AIM

To continue the development of counting skills and to deepen the realization of pattern and order in the number system.

NOTES

1. Related parts from earlier sections of the Guide should be read in conjunction with this topic. These can be found from the index in Section F.

2. Work outlined in this part presents one particular area of the course. However, this work should not be treated separately and in isolation, but should be integrated with other areas in Section G.

3. Regular and frequent oral work should be taken in this topic, either in group or individual counting or in discussion of written activities.

DEVELOPMENT

Note : Teachers are not committed to treat this work in the sequence shown.

1. Counting of Numbers, Forwards or Backwards, in Sequences, with an Upper Limit of One Million

(a) *By ones.* Activities suggested for Section F can well be used here. Counting ranges should be limited and chosen to treat particular difficulties.

(b) *By twos, by threes, by fours, and by all of the whole numbers to twelves.* Counting may be from zero or from any other number within the counting range. Children should be encouraged to look for and utilize final digit patterns.

Example : 1, 5, 9, 13, 17, 2<u>1</u>, 2<u>5</u>, 2<u>9</u>, 3<u>3</u>, 3<u>7</u>, 41, — — —

Activities using the number chart as suggested in the Guide, Section F, pages 7–9, are useful in this section also.

(c) *By tens, by twenties, by thirties, and so on to nineties.* Here the attention is directed to the tens digit.

(d) *By hundreds and by thousands.* Here the objective of counting activities is to establish and/or to reinforce ideas about place value in the number system.

2. Further Study of Patterns in Number Sequences

Here the following ideas are investigated further :

(a) Odd and even numbers ;

(b) doubling and halving techniques ;

(c) serial addition and subtraction ;

(d) addition or subtraction of both constant differences and changing differences ;

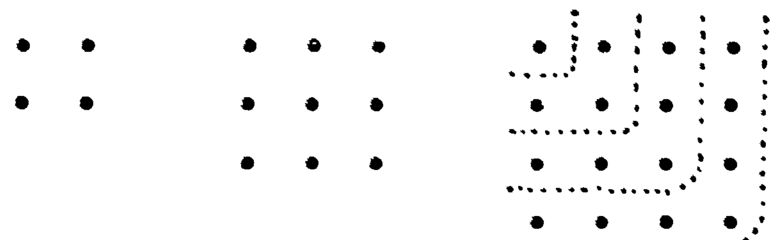
(e) increase or decrease in a sequence by common ratio

(f) squaring.

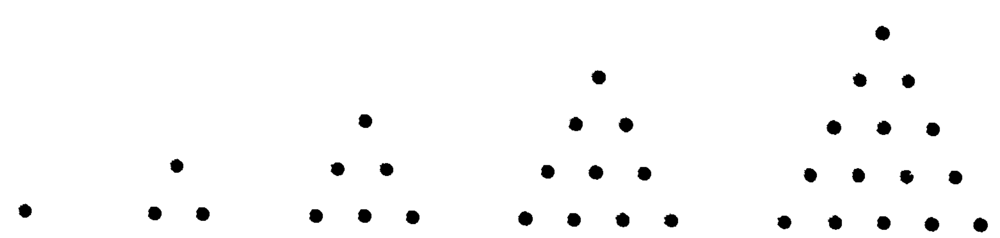
Odd and even numbers.—Interesting and profitable discoveries that can be made from a study of these numbers are considered in the Guide to Section F (page 9). To these can be added a simple study of both square numbers and triangular numbers. For example :

(i) The sum of consecutive odd numbers, commencing with 1, is always a square number—

$1 + 3 = 4$; $1 + 3 + 5 = 9$; $1 + 3 + 5 + 7 = 16$.



(ii) The sum of successive pairs of triangular numbers is a square number—



1 3 6 10 15

$1 + 3 = 4$; $3 + 6 = 9$; $6 + 10 = 16$; $10 + 15 = 25$; and so on.

It is important that such facts be "discovered" by the child through experience gained generally through specially designed activities.

Doubling and halving.—The skill developed in Section F should be maintained. Regular practice should be given in halving the single digits—1, 2, 3, · · · 9. Occasionally larger numbers, both odd and even should be used. Exercises such as the following are suitable :

(i) 3, $1\frac{1}{2}$, *, *, *, $\frac{3}{32}$.

(ii) 9, *, *, *, *, $\frac{9}{32}$, $\frac{9}{64}$.

(iii) 72, *, 18, 9, *, *, *, $\frac{9}{16}$.

Encouragement should be given to the child to investigate the possibilities of doubling and halving in equation work (see Guide to Section F, page 11). Examples employing this technique are :

(i) $35 \times 16 = \square$	(ii) $256 \div 16 = \square$	(iii) $288 \div \square = 24$
$70 \times 8 = \square$	$128 \div 8 = \square$	$144 \div \square = 12$
$140 \times 4 = \square$	$64 \div 4 = \square$	$72 \div \square = 6$
$280 \times 2 = \square$	$32 \div 2 = \square$	$36 \div \square = 3$
$560 = \square$	$16 = \square$	

Serial addition and subtraction.—These processes are adequately covered in the Guide to Section F. Ideas and skills developed there should be maintained through regular practice involving both oral and written activities.

The study of sequences of numbers should be continued and extended. In addition to the recognition of odd and even numbers, serial addition or subtraction, and doubling and halving, the child should recognize pattern in a sequence involving addition or subtraction of constant or changing differences, the increase or the decrease in sequences being by common ratios or by squaring. Examples of sequences should provide sufficient terms to suggest a possible pattern and to check the pattern. For example :

(i) 2, 5, 8, *, *, *, *, 23.

(ii) 56, 49, *, *, *, 21.

(iii) ·7, ·8, *, *, 1·1.

(iv) $\frac{5}{8}$, $\frac{7}{8}$, *, *, *, $1\frac{7}{8}$.

(v) $1\frac{1}{2}$, 3, *, *, *, 9.

(vi) 2, 3, 5, 8, *, *, *, 30.

(vii) 1, $1\frac{1}{2}$, *, 4, 6, $8\frac{1}{2}$, *.

(viii) *, 4, 9, *, *, *, 49.

(ix) *, $\frac{1}{9}$, $\frac{1}{16}$, $\frac{1}{25}$, *, $\frac{1}{49}$.

(x) 1, 7, 2, 14, 3, *, *, *, 5.

The exercises listed above cover a wide range of difficulty. Many examples of each type should be discussed. Other types may be used, especially when they assist in the development of a particular topic.

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Number patterns can be extended to arrays such as :

(i)

1	3	5	7	9	11	13	15
2	6	10	14	18	22	26	30
4	12	20	28	36	44	52	60
8	24	40	56	72	88	104	120
16	48	80	112	144	176	208	240

The numerals in heavy type only are given. The empty cells are to be filled.

The child should make a practice of checking his first estimate of the pattern by checks built into the exercise. One pair of numbers is never sufficient to establish a given or an intended pattern.

(ii)

1	3	5	7	9	11
5	7	9	11	13	15
$2\frac{1}{2}$	$3\frac{1}{2}$	$4\frac{1}{2}$	$5\frac{1}{2}$	$6\frac{1}{2}$	$7\frac{1}{2}$
1	2	3	4	5	6

The numerals in heavy type only are given.

In patterns such as the above, relationships are built in, both in the rows and in the columns. Some diagonal patterns may also become evident to the child ; indeed the discovery of these should be encouraged.

Arrays such as the above can be constructed by writing a chosen row pattern, for example, odd numbers as in the above example. Selected operations may then be carried out on these, say, as in (ii) above—adding 4, halving, and then subtracting $1\frac{1}{2}$. Certain numbers are then selected to provide sufficient information for the child to complete the array. Exercises such as these should increase the child's awareness of pattern in the number system ; they also assist in the reinforcement of number facts.

The child should be given the opportunity to create such patterns for other class members to solve. In such activities the number board is a useful aid.

The child will need many more cues for the recognition of patterns in the earlier stages than in the later ones. Eventually, only the absolute minimum need to be given. At the outset, complete patterns could be presented and, from discussion, the operations carried out or the relationships used could be discovered.

PLACE VALUE

AIM

To extend the study of the decimal system of numeration with its principles of place value to one million, including numerals in decimal form for both tenths and hundredths.

NOTES

1. On the completion of Section F the child is able to—

(i) analyse in detail numbers to thousands in terms of place value ;

(ii) recognize, read, and write numbers to 1,000,000 ;

(iii) record in decimal notation numerals corresponding to vulgar fractions with denominators of ten.

2. The aim of this part is to extend the detailed study of place value to millions and to hundredths.

It is of historical interest to note that various forms of notation have been used to distinguish the fractional part of a number. Even today the form of notation for decimal fractions has not been standardized throughout the world. In general, however, decimal fraction notation is simply an extension to the right of the units place for the system of notation. When recording numbers it is essential to locate the units place. The decimal point assists in this regard.

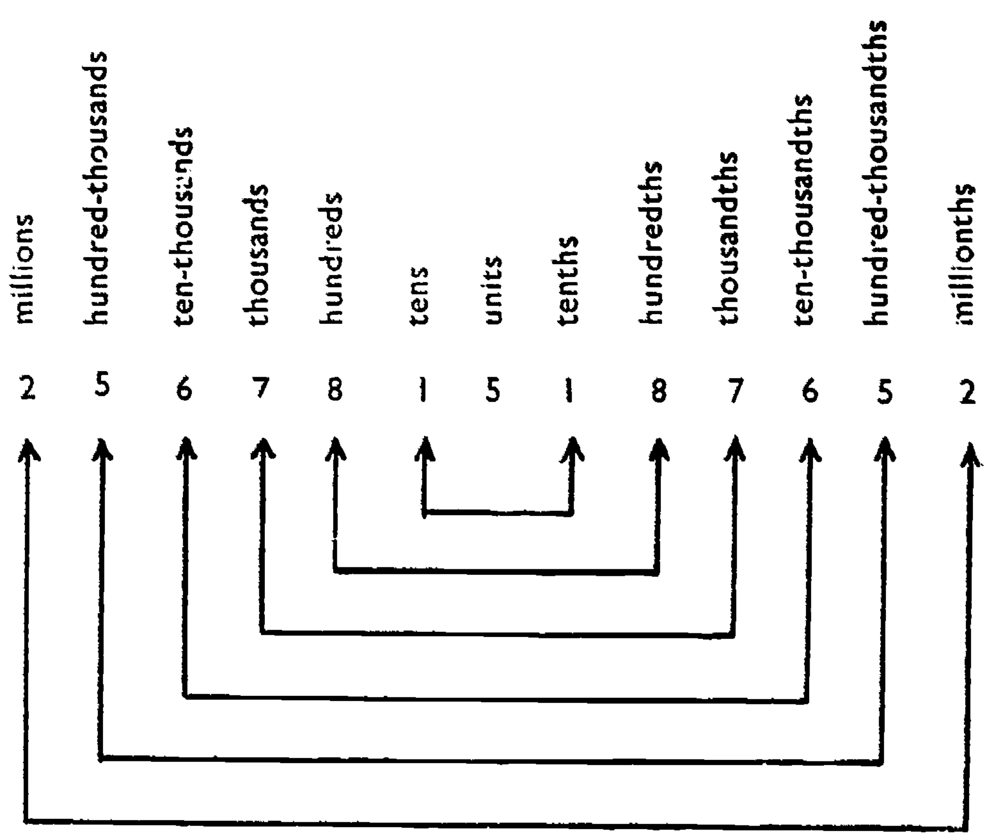
The value of a digit in a numeral depends on the digit itself (0, 1, 2, . . . 9). It also depends on the place the digit occupies in the numeral (whether "tens" or "tenths", and so on). For example, in the numeral 526, the 5 written in the third place signifies in itself the same thing as five tallies, such as / / / / /, while by virtue of its position it also means 100s. These two meanings, taken together as 5 x 100, cause the 5 to contribute 500 to the total number represented by 526.

Similarly, in the fraction ·25, the 5 has the value five times the value $\frac{1}{100}$, or a total value $\frac{5}{100}$.

The diagram on page 12 illustrates the run of the decimal places both to the right and to the left of the centre of the system of notation—the units place.

It can be seen from the diagram that the tens place is the first place to the left of the units and that the tenths place is the first place to the right of the units place. Millionths are the same number of places to the right of the units place as millions are to the left.

3. In this section the child's understanding of the order and values of numbers to 1,000,000 is developed mainly through the work in place value and numeration and notation. Some oral work in counting is still necessary to ensure that the child knows the sequence of number names



from 1 to 1,000,000 and from 1 to ·09. Oral counting should be limited to a specified range ; for example, " Start at 975,491 and count back to 975,482 by ones." " Start at ·2 and count by ' two-tenths ' to 2·4."

4. Before extending the study of place value to the right of the units place to include tenths and hundredths, it is essential that the child understands place value in terms of whole numbers to tens of thousands at least. It must again be emphasized that understanding must be established through oral work, and notational skills must be acquired before the child is asked to operate with large numbers or to analyse in detail the place value of digits in numerals representing large numbers.

5. Decimals should not be read in an abbreviated form until the child has a thorough understanding of place value to hundredths. For example, 10·8 should not be read as " one nought point eight " or " ten point eight ", which can be meaningless. Since the aim is to understand the system of notation and place value, all readings of decimals should be in full ; that is, the numerator and the denominator are named as in reading a common fraction. For example, 10·8 is read as " ten and eight-tenths ".

6. The activities outlined in the development below are extensions of basic mathematical ideas studied in previous sections and illustrate a variety of approach. The order in which these activities should be treated is not important. It is important, however, that all should be covered so that the child is given a wide experience.

DEVELOPMENT

Reading Large Numbers

Before an intensive study of place value of numbers to 1,000,000 is undertaken, the teacher must ensure that the child is able to—

(i) read and write numbers to 1,000,000 ;

(ii) group the digits of a numeral into sets of three, starting at the units place ;

(iii) identify the place names of the digits in a numeral.

The exercises listed below emphasize the basic technique of notation. (This step is an extension of the work outlined in Curriculum Guide, Pure Number, Section F, Part 2, Development. This guide should be referred to throughout the development of this topic.)

(i) Name the places in which the digits are written in the numeral 1,364,275.

(ii) Read the numeral 17,717.

(iii) Write the numeral for : " Four hundred and sixty thousand, seven hundred and fifty-two."

(iv) Rewrite the numeral 853,741 in extended notation. (Refer to Curriculum Guide, Section F, page 16.)

Renaming Numbers

Activities involving the renaming of numbers stress the various relations within the system of notation. The following examples illustrate a sequence of development for the main ideas. (Reference should be made to the Curriculum Guide, Section F, pages 16 and 18.)

Rename 60,000

(i) 60,000 = 6 x 10,000
= 6 x (10 x 1,000)
= (6 x 10) x 1,000 (Associative law)
= 60 x 1,000

Then 60,000
= 6 x 10,000
= 60 x 1,000

(ii) If the 1,000 in 60 x 1,000 is renamed as (10 x 100)
then 60 x 1,000 = 60 x (10 x 100)
= (60 x 10) x 100 (Associative law)
= 600 x 100

Then 60,000 can be renamed as :
6 x 10,000
60 x 1,000
600 x 100

(iii) If the 100 in 600 x 100 is renamed as (10 x 10)
then 600 x 100 = 600 x (10 x 10)
= (600 x 10) x 10 (Associative law)
= 6,000 x 10

Then 60,000 can be renamed as :

$$6 \times 10{,}000$$
$$60 \times 1{,}000$$
$$600 \times 100$$
$$6{,}000 \times 10$$
$$\square \times \square$$

From a study of the above recordings a visual pattern will be discovered by the child. A similar pattern was encountered by the child when he was doubling and halving in previous sections. This exercise illustrates the fact that we are multiplying and dividing successively by 10 in the same way as we multiplied and divided successively by 2 when we doubled and halved. For example, the child was asked to study the following equations, name the pattern, and write the next two equations :

$$4 \times 6 = 24$$
$$2 \times 12 = 24$$
$$1 \times 24 = 24$$
$$(\tfrac{1}{2} \times 48 = 24)$$
$$(\tfrac{1}{4} \times 96 = 24)$$

The teacher should continually ask the question : "What is the pattern ?"

The numeral used in examples (i) to (iii) above has one non-zero digit, namely "6". Treatment is now extended to numerals with two non-zero digits, such as 25,000.

(i) 25,000 = 2 ten-thousands + 5 thousands
= 25 thousands
= 25 thousand units
= 250 hundreds
= 2,500 tens

A further extension is now made to numerals with more than two non-zero digits.

(ii) 426,315 = 4 hundred-thousands + 2 ten-thousands + 6 thousands + 3 hundreds + 1 ten + 5 units.
426,315 = 42 ten-thousands + 6 thousands + 3 hundreds + 1 ten + 5 units.
426,315 = 426 thousands + 3 hundreds + 1 ten + 5 units.
426,315 = 425 thousands + 13 hundreds + 15 units.
426,315 = 400,000 + 20,000 + 6,000 + 300 + 10 + 5.

(iii) Write four different numerals for each of these numbers : 15,641 ; 623,330.

(iv) Rewrite in digits each of the following : Eighteen thousands + three hundreds + eleven units.

Forty-seven ten-thousands + eight thousands + ninety-four tens + six units.

Four ten-thousands + twenty-eight hundreds + fifty-three units.

(v) Write each of the following in digits :

365 tens + 7 units.

46 hundreds + 8 tens + 4 units.

Patterns

The child should realize that the number system is not simply a haphazard collection of figures, but is highly organized, self-consistent, complex, and interrelated.

A study of place value requires an understanding of sequences such as the following :

"Write the next two terms in the following sequences" :

(i) 3; 30 ; 300 ; 3,000 ; ——— ; ———.

The child should be able to express the relationship in words as well as in digits. That is, he should be able to say that the next two terms are 30,000 and 300,000 because each term in this sequence is ten times greater than the one which precedes it.

(ii) 100,000 ; 10,000 ; 1,000 ; ——— ; ———.

In this sequence the next two terms are 100 and 10 because each term is one-tenth of the term that precedes it.

Extension of Study of Place Value

A suggested treatment is as follows :

(i) The following pattern is built up in stages during oral work and recorded by the teacher on the blackboard so that a visual pattern is presented to the child.

3
13
213
5,213
75,213
675,213
4,675,213

The teacher writes the numeral 3 on the blackboard and asks the following questions :

"What number does the numeral 3 stand for ?" (Three.)

"In what place is 3 written ?" (The units place.)

The teacher then writes the digit 1 to the left of the numeral 3.

"What number does this represent ?" (Thirteen.)

"How many digits in the numeral 13 ?" (Two.)

"How many places are there ?" (Two.)

"Name these places." (Units place and tens place.)

"Which digit is written in the units place ?" (3.)

"Which digit is written in the tens place ?" (1.)

"Write 13 in extended notation."

The teacher then writes the digit 2 to the left of the numeral 13.

"How many digits in the numeral 213 ?" (Three.)

"How many places ?" (Three.)

"In which place is the 2 written ?" (Hundreds place.)

"What number does 213 represent ?" (Two hundred and thirteen.)

"Write 213 in extended notation."

This procedure should be continued to 4,675,213.

(ii) The teacher writes the numeral 1,111 on the blackboard and asks :

"How many digits in this numeral ?" (Four.)

"How many places are there in this numeral ?" (Four.)

"What can you tell me about the pattern of digits in this numeral ?" (The same digit is repeated.)

"Does each digit represent the same number ?" (No.)

"Why ?" (Because they are written in different places in the numeral.)

"What number does the numeral 1,111 name ?" (One thousand, one hundred and eleven.)

"Write the numeral 1,111 in extended notation."

$(1{,}111 = 1{,}000 + 100 + 10 + 1)$

$(1{,}111 = 1 \times 1{,}000 + 1 \times 100 + 1 \times 10 + 1 \times 1)$

"How many times greater is the 1 in the tens place than the 1 in the units place ?" (Ten times greater.)

"Write this as an equation." $(10 = 10 \times 1)$

"How many places to the left of the 1 in the units place is the 1 in the tens place ?" (One place to the left.)

"What fraction is the value of the 1 in the units place of the value of the 1 in the tens place ?" (One-tenth.)

"Write this as an equation." $(1 = \frac{1}{10} \times 10)$

"What is the value of the 1 written in the fourth place ?" (One thousand.)

"What is the value of the 1 written in the second place ?" (One ten.)

"How many places to the left of the tens place is the thousands place ?" (Two.)

"How much greater is the value of the 1 in the thousands place than of the 1 written in the hundreds place ?" (Ten times greater.)

"Write this as an equation." $(1{,}000 = 10 \times 100)$

"How many places to the right of the thousands place is the tens place ?" (Two.)

"What fraction is the value of the 1 in the tens place of the value of the 1 in the thousands place ?" (One-hundredth.)

"Write this as an equation." $(10 = \frac{1}{100} \times 1{,}000)$

The above procedure is repeated to study numerals such as 222,222 ; 55,555 ; 7,777.

The child must master this step before it can be said that he has a sound understanding of the system of numeration and the relations that exist within it.

Decimal Notation—Extensions to Hundredths

Reference should be made to the Curriculum Guide, Pure Number, Section F, pages 18 and 19.

A procedure, similar to that for introducing tenths, may well be used to introduce hundredths. The child is aware of the relative values of the digits as placed in the numeral 11·1. The continuation of this pattern of relationships will assist him to name the next place as the "hundredths" place. Then the values of the digits in 111·11 can be called one hundred, one ten, one unit, one tenth, one hundredth. Numerals such as 2·22 and 88·88 can be treated similarly. When the "hundredths" place is accurately recognized on a number of occasions, variations can be introduced. For example, the digit in the tenths place can vary from that in the hundredths place. "Zero" in the tenths place may be used. Numerals may be broken down, reordered, and reassembled.

Thus :

seven tenths + 3 hundreds + 9 hundredths = $\boxed{300{\cdot}79}$

Activities similar to those suggested for whole numbers can well be used with decimals.

Counting activities are important. The staircase of rods is useful. Orange can be designated as $\frac{1}{10}$, thus white has the value $\frac{1}{100}$. Using the staircase, the difficult hurdle from $\frac{9}{100}$ to $\frac{1}{10}$ can be surmounted fairly easily in most cases. (From understandings of place value patterns and of equivalence of fractions, and from experience with graph paper ruled in ten divisions to the inch or with other structured aids, or from a combination of these, the child will gain understanding that $\frac{1}{10} = \frac{10}{100}$.)

Counting activities such as the following should be regularly taken.

(i) $\frac{7}{100}$, $\frac{8}{100}$, $\frac{9}{100}$, $\frac{1}{10}$, $\frac{11}{100}$, $\frac{12}{100}$.
(ii) $\frac{88}{100}$, $\frac{89}{100}$, $\frac{9}{*}$, $\frac{91}{100}$, $\frac{92}{100}$.
(iii) ·89 , ·9 , * , * .
(iv) 1·87 , 1·88 , * , * , * .
(v) 9·95 , 9·96 , * , * , * , * , * , 10·02.

Activities to reinforce the understanding of decimal notation to hundredths can involve the use of :

Cuisenaire rods—as discussed above ;
Dienes's base 10 Multibase Arithmetic Blocks ;
area diagrams—units ruled to form 100 small squares ;
dollars and cents ;
the number line ;
the abacus ;
extended notation.

Hence the following activities :

(i) Given that the orange rod has the value $\frac{1}{10}$, write down the value of each of the following :

(a) black ;

(b) black + yellow ;

(c) nine orange rods + blue + red.

(ii) Using graph paper (ten rulings to the inch) and giving the inch square the value 1, write values for :

(a) 3 inch squares and 29 hundredth squares ;

(b) 2 inch squares + 125 hundredth squares.

(iii) (a) Write \$12·09 in dollars and cents.

(b) Write 13 dollars 93 cents in decimal form.

(c) How much will I have when 2 cents are added to \$3·99 ?

(iv) Mark each of the following on a number line :

(a) 1·1

(b) ·99

(c) 1·19

(v) Rewrite the following numerals in simple decimal form :

(a) 9 tenths + 7 hundredths ;

(b) 8 hundredths + 3 units ;

(c) 347 hundredths ;

(d) 15 tens + 15 hundredths ;

(e) 5 hundredths + 7 tenths ;

(f) 23 hundredths + 6 units.

BASIC PROPERTIES

AIM

To reinforce and extend the child's understanding of the basic properties of numbers learned in previous sections.

NOTES

1. The *basic properties* of numbers are sometimes referred to as the *mathematical laws* of number. In this Guide the two terms, wherever used, can be regarded as interchangeable.

2. The work in Section G is an extension of that in Section F. It is important to consult Part 4 of that section in the Curriculum Guide (Pure Number) for a proper understanding of the following :

(i) The implications of the properties for teaching ;

(ii) the nature of these properties or laws ; and

(iii) the activities used for introducing them in the classroom.

3. To attain a full understanding of these properties or laws, it is necessary for the child to understand both the *inclusive* and the *exclusive* nature of each property. That is to say, it is necessary to know both where the property can be used validly and where it cannot. For instance, the commutative and the associative properties apply to addition and multiplication, but they do not apply to the processes of subtraction and division.

4. The study of the distributive property is now extended to cases of multiplication over several addends.

5. The distributive property of multiplication over subtraction should become known.

For example :

$$\begin{aligned} 280 \times 5 &= (300 - 20) \times 5 \\ &= (300 \times 5) - (20 \times 5) \end{aligned}$$

6. This part of the course should not be taught in isolation from the other parts of this section. Ideas from it should be used, where appropriate, when treating computation exercises in the four basic processes applied to whole numbers. There will also be computation exercises in applied number in which a knowledge of the basic properties will suggest a method of attack. The activities that follow these notes give some idea of the place of the basic properties in this section.

7. The child should come to regard the basic properties of numbers as useful to assist in understanding a question and carrying out a computation. In Section G, the basic properties are a means to an end rather than an end in themselves.

ACTIVITIES

1. Once a child has solved a given equation he may be asked to explain both the how and the why of what he did. For example, he may be asked to solve :

(i) 4 hours 20 minutes + 2 hours 35 minutes = $\boxed{}$

In doing this he may first add 4 hours to 2 hours and then add the 20 minutes to the 35 minutes. The child's oral explanation for this procedure when written down may appear as :

4 hours 20 minutes + 2 hours 35 minutes

(4 hr. + 20 min.) + (2 hr. + 35 min.) (Renaming)

4 hr. + (20 min. + 2 hr.) + 35 min. (Associative property)

4 hr. + (2 hr. + 20 min.) + 35 min. (Commutative property)

(4 hr. + 2 hr.) + (20 min. + 35 min.) (Associative property)

6 hr. + 55 min.

6 hours 55 minutes.

(ii) $3 \times 213 = \square$

$$\begin{aligned} 3 \times 213 &= 3 \times (200 + 10 + 3) && \text{(Renaming)} \\ &= (3 \times 200) + (3 \times 10) + (3 \times 3) && \text{(Distributive property)} \\ &= 600 + 30 + 9 \\ &= 639 \end{aligned}$$

(iii) 3 gal. 2 pints x 3 = (3 gal. + 2 pints) x 3 (Renaming)
= (3 gal. x 3) + (2 pints x 3) (Distributive property)
= 9 gal. + 6 pints
= 9 gallons 6 pints

(iv)
$$\begin{aligned} 94 + 63 &= (90 + 4) + (60 + 3) && \text{(Renaming)} \\ &= 90 + (4 + 60) + 3 && \text{(Associative property)} \\ &= 90 + (60 + 4) + 3 && \text{(Commutative property)} \\ &= (90 + 60) + (4 + 3) \\ &= 150 + 7 \\ &= 157 \end{aligned}$$

2. An activity such as the following provides discussion points for children in considering where the various basic properties do and do not apply.

(i) Complete each of the following equations : (The first example has been done for you.)

$74 + 21 = \boxed{95}$

$24 \times 3 = \square$

$$17 + 28 = \square$$
$$29 - 17 = \square$$
$$32 \div 8 = \square$$

(ii) Now rewrite each of the equations by changing only the order of the numbers on the left side.

(iii) Which of the rewritten equations are true ?

3. Without computation, complete each of the following sentences correctly by using either $=$ or $\neq$ in the space marked $\triangle$:

(The first example has been completed for you.)

$$49 \div 7 \neq 7 \div 49$$
$$59 + 72 \ \triangle \ 72 + 59$$
$$52 \times 8 \ \triangle \ 8 \times 52$$
$$78 - 27 \ \triangle \ 27 - 78$$

4. Complete the following equations in the second column that can be solved without calculation by using the information given in the first column :

(The first example has been completed for you.)

Table

Ist *Column*	2nd *Column*
$478 + 273 + 172 = 923$	(*a*) $172 + 273 + 478 = \boxed{923}$
$394 \times 8 = 3152$	(*b*) $29 \times 65 = \square$
$470 - 132 = 338$	(*c*) $132 \div 7 = \square$
$924 \div 7 = 132$	(*d*) $\square = 470 - 132$
$92 \times 65 = 5980$	(e) $(394 \times 8) + (394 \times 1) = \square$
	(*f*) $65 \times 92 = \square$
	(*g*) $874 + 372 + 271 = \square$
	(*h*) $(394 \times 4) + (394 \times 4) = \square$

5. Use brackets where necessary to make each of the following equations true :

(i) $12 - 8 - 4 = 8$
(ii) $32 \div 8 \div 2 = 8$
(iii) $17 - 8 - 3 = 6$
(iv) $35 + 17 + 41 = 93$
(v) $27 \div 9 \div 3 = 9$

21

EQUATIONS

AIM

To consolidate and extend the child's understandings of, and abilities in—

(*a*) the reading and interpretation of equations ;
(*b*) the manipulation of equations ;
(*c*) creative work with equations ;
(*d*) the solution of equations.

NOTES

The Course of Study provides for " maintenance of Section F " in the aspects (*a*), (*b*), (*c*), and (*d*) above. Consequently the Curriculum Guide, Pure Number, Section F, Part 5, Equations, should be read in conjunction with this part of the present Curriculum Guide.

The aspect of the course dealing with equations arises incidentally when treating such matters as doubling and halving numbers, place value, the basic properties, and interrelationships of operations. This aspect is not to be treated in isolation. Consequently, there is no separate *development* of this topic as such ; *activities* arise incidentally in the context of the other aspects of pure number, such as doubling and halving, mentioned above. Further activities arise naturally in several aspects of applied number.

Example 1. 2 lb. 4 oz. x 8 = $2\frac{1}{4}$ lb. x 8.

Example 2. 2 lb. 11 oz. + 1 lb. 7 oz. = 2 lb. + 11 oz. + 1 lb. + 7 oz.

Example 3. \$25·12 ÷ 16 = 2512 cents ÷ 16.

Example 4. 9 tons + 38 cwt. = 9 tons + 1 ton +18 cwt. = 10 tons 18 cwt.

Order of Operations

As early as Section B (see Guide, p. 35) the following situation is treated :

$$6 - 5 = 1$$

Substituting 3 + 2 for 5, and holding " to our original intention ",

$$6 - (3 + 2) = 1.$$

If the brackets were not present, then $6 - 3 + 2$ might well be read as $3 + 2$, or 5, and clearly 5 is *not* equal to 1 ; that is,

$$(6 - 3) + 2 = 5, \quad \text{while} \quad 6 - (3 + 2) = 1.$$

We cannot be sure whether

$$6 - 3 + 2 = 5 \quad \text{or} \quad 6 - 3 + 2 = 1$$

unless *either* brackets are used to make clear the intention about the order of adding and subtracting *or* there is some definite rule or convention about the order of operations to guide us.

From this discussion, three important points arise.

First.—If there is doubt as to the order in which mathematical operations must be carried out, then brackets should generally be used to make our intention clear.

Example 1 : $12 \div (3 \times 4) = 1$, but $(12 \div 3) \times 4 = 16$.
Example 2 : $(9 - 5) - 4 = 0$, but $9 - (5 - 4) = 8$.
Example 3 : $4 + (6 \div 2) = 7$, but $(4 + 6) \div 2 = 5$.

Second.—If the order of carrying out operations does *not* affect the outcome, then brackets need not be used.

Example 1 : $7 + (3 + 2) = (7 + 3) + 2$ (Associative law of addition)
Example 2 : $7 + (3 - 2) = (7 + 3) - 2$.
Example 3 : $12 \times (6 \times 2) = (12 \times 6) \times 2$ (Associative law of multiplication)
Example 4 : $12 \times (6 \div 2) = (12 \times 6) \div 2$

In each of these four examples, the brackets may well have been omitted.

Third.—There are equations in which the order of carrying out the operations might affect the outcome, but the intention seems clear without the over-use of brackets.

Example : In a case of repeated subtraction, a child might write

$$12 - 3 - 3 - 3 - 3 = 0 \quad . \; . \; . \; . \; . \; . \; . \; . \quad \text{(i)}$$

Strictly speaking, however, he should write

$$(((12 - 3) - 3) - 3) - 3 = 0$$

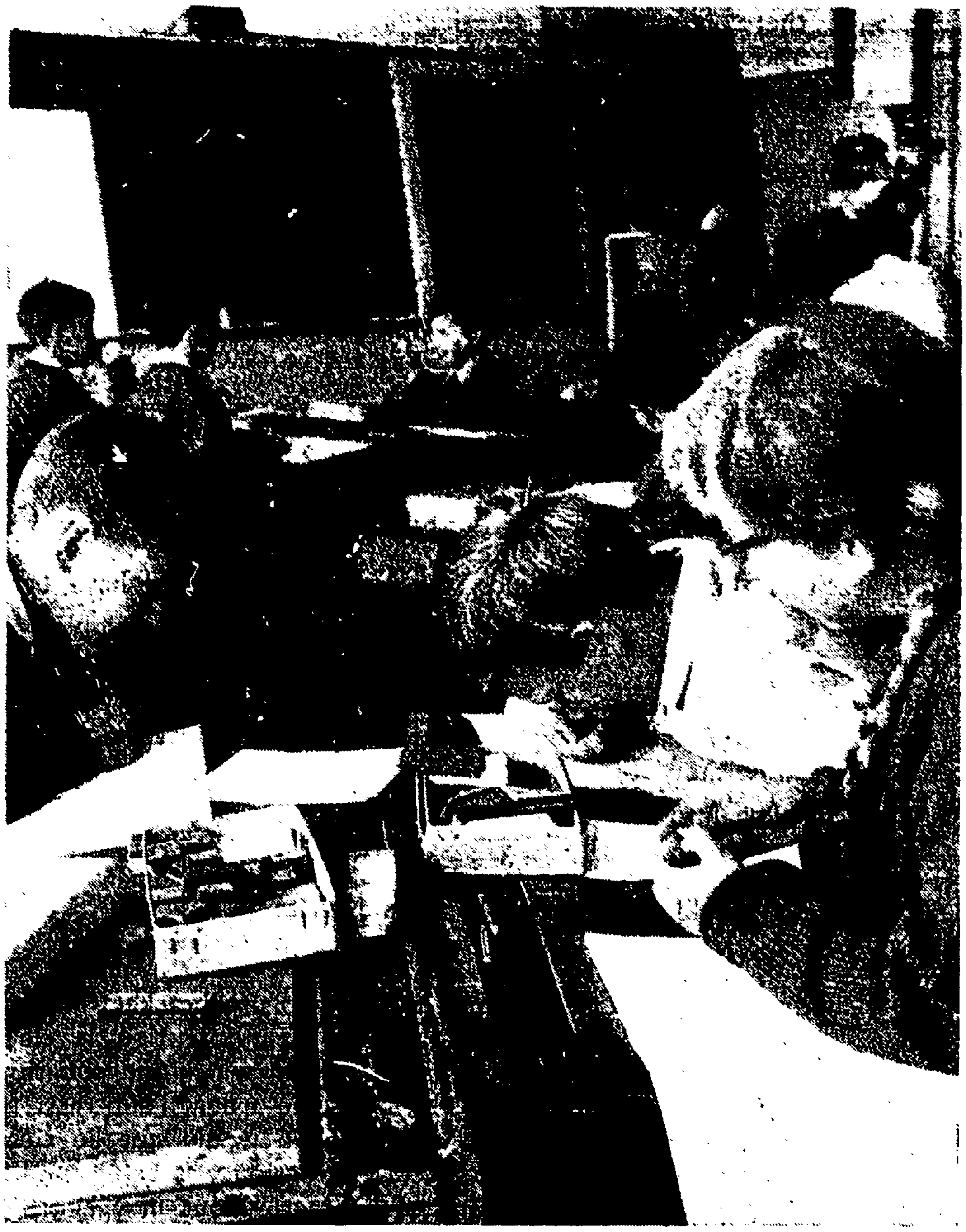
Children and teachers are most unlikely to use brackets in such an example because they believe their intention is clear and because of the cumbersome way in which the equation is written with brackets. However, the order of subtractions on the left side of equation (i) above would not be clear unless it was understood that a convention was being used—that the subtractions were to be carried out in the order in which they are read, from left to right.

In the absence of brackets, the following convention should be followed regarding the order of operations :

If several operations occur in succession, first multiplications and divisions are carried out, then additions and subtractions, and in the order in which they occur.

Example :
$$\begin{aligned} 7 - 8 \div 4 \times 2 + 3 &= 7 - 2 \times 2 + 3 \\ &= 7 - 4 + 3 \\ &= 6. \end{aligned}$$

In passing, it is interesting to note that this convention is used in *ALGOL* and *FORTRAN*, "languages" used in programming digital computers.



In conclusion, however, it should be stressed that brackets should be used if there is likely to be a doubt as to the order in which operations are to be carried out.

"Of" as a Mathematical Operation

Expressions such as "$\frac{1}{2}$ of 7" occur commonly. By the time pupils have reached Section G of the course they should not be set exercises by the teacher in which "of" occurs as a formal operation with a special precedence and meaning of its own. A comment on "of" and its occurrence in the teaching of this course will be found in *Course of Study for Primary Schools, Mathematics*, 1967, *Pure and Applied Number, Sections G, H, I*, page 6.

INTERRELATIONSHIPS OF OPERATIONS

AIMS

1. To further consolidate and extend the child's understanding of the four basic operations.

2. To use this understanding to facilitate the solution of problems.

NOTES

1. The child has had some experience with all the relationships to be considered in this section since Section C, and reference should be made to Curriculum Guides, Pure Number, Sections B to F, for the developments of the topics.

2. At the outset, the child should review the situations in terms of small, whole numbers. For example, consider the number triple (6, 3, 18). Arising from this we have $6 \times 3 = 18$; $18 \div 3 = 6$; $18 \div 6 = 3$. As children near the end of Section G, examples such as the following should be understood :

$$61 \times 12 = 732\ ;\ 732 \div \square = 61\ ;\ 732 \div 61 = \square.$$

To test understanding of this interrelationship an example such as the following could be used :

If $479 \times 742 = 355{,}418$

then $355{,}418 \div 742 = \square$.

(*Note* : No computation is involved.)

3. A thorough understanding of relationships will enable the child to solve equations without the necessity for recourse to meaningless rote procedures. Although eventually it will appear that the child is using rote procedures in simplifying equations, he should be able, if called upon to do so, to justify his actions.

DEVELOPMENT

The various interrelationships are here discussed in isolation and in sequence. In practice, however, no such compartmentalization is either necessary or desirable. Very much of the development is likely to be incidental and frequent opportunities should be contrived to keep the ideas constantly before the child.

Where a lack of understanding occurs in respect to one or more of the relationships concerned, it is important to remedy this before proceeding to formal algorithms where the relationships are important. Thus subtraction should be understood as the inverse of addition, multiplication as successive addition, division as the inverse of multiplication and, in certain situations, as successive subtraction.

Addition and Multiplication (Multiplication as Repeated Addition)

Reference should be made to Curriculum Guide, Pure Number, Section B, Stages 8, 9, and 11 ; Section E, Stage 30 ; and Section F, pp. 89, 90.

The teacher should assure himself that this idea is thoroughly understood. Children should confidently solve equations such as :

(i) $7 + 7 + 7 = 3 \times \square$

(ii) $247 + 247 + 247 + 247 = \triangle \times \square$

(iii) $428 \times 3 = \square + \square + \square$

Subtraction and Division (Division as Repeated Subtraction)

Reference should be made to the Curriculum Guide, Pure Number, Section B, p. 35 ; Section C, p. 126 ; Section E, p. 30, and Section F, pp. 98–100.

A thorough understanding of this idea is a most important prerequisite for the understanding of the division algorithm as developed in the course. This is true, too, for the reduction of compound quantities from smaller to larger units of measure.

Activities in the further development of this topic can involve—

(i) the number line ;

(ii) discrete materials such as grains of wheat, marbles, or other counters ;

(iii) quantities of material such as ounces of sand, inches of string, or pints of water.

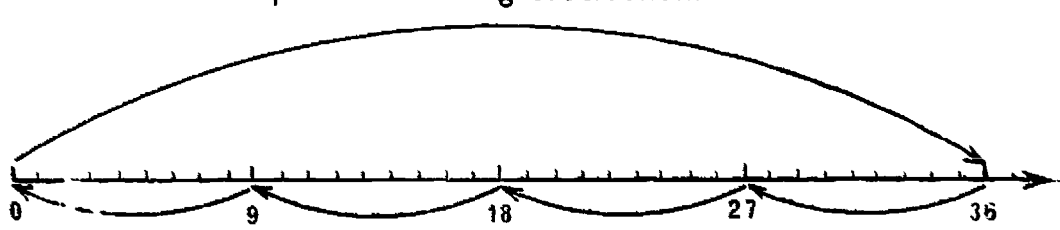
Exercises of the following types will be found useful :

(*a*) (i) $36 - 9 - 9 - 9 - 9 = \square$

(ii) $36 - (4 \times \triangle) = 0$

(iii) Solve $36 \div 9 = \square$ by subtraction .

(iv) Use the information shown in the diagram below to write an equation involving subtraction.



(v) $76 \div 19 = \square$ (Solve by subtraction.)

(vi) $55 \div 9 = \square$ (Solve by subtraction.)

(*b*) A bag contains 68 marbles.

(i) How many times can I take 17 marbles from the bag ?

(ii) How many 17s are there in 68 ?

(iii) Rewrite (ii) (above) as an equation, using first the subtraction operation and then the division operation .

Note : Activities of this type, where recourse to actual material is possible, may assist the child who had previously not fully understood the relationship between subtraction and division using other types of material.

(c) (i) How many pieces of string, each one foot long, can be cut from a length of 80 inches ?

(ii) How many pieces of plastic tubing, each one yard long, can be cut from a roll containing 240 inches ?

Inverse Operations

Inverse operations (multiplication—division ; addition—subtraction) have been used by the child since Section C, particularly in Section F.

Through " pattern " activities the child should achieve the following understandings in Section G.

(i) $\square - \square = 0$ and (ii) $\square \times \frac{1}{\square} = 1$.

Note : (1) $\square \div \square$; $\square \times \frac{1}{\square}$; and $\frac{\square}{\square}$ are equivalent.

(2) The convention in regard to the use of frames, both here and throughout Section G, is as set out in Curriculum Guide, Pure Number, Section F, page 104 (see paragraph beginning : "Another widely used and accepted convention").

(3) The number " 0 " is called the neutral or identity element for addition and the number " 1 " is the neutral or identity element for multiplication for the numbers of arithmetic.

(4) $\square \times \frac{1}{\square}$ is meaningless if 0 is written in place of $\square$.

Exercises such as the following could well be used to establish the desired generalizations :

(a) *Subtraction as the Inverse of Addition*

(i)
$20 - 4 = \square$
$10 - 4 = \square$
$9 - 4 = \square$
$8 - 4 = \square$
$7 - 4 = \square$
$6 - 4 = \square$
$5 - 4 = \square$
$4 - 4 = \square$

(ii)
$30 - 7 = \square$
$20 - 7 = \square$
$10 - 7 = \square$
$9 - 7 = \square$
$8 - 7 = \square$
$7 - 7 = \square$

(iii)
$30 + 7 - 1 = \square$
$30 + 7 - 2 = \square$
$30 + 7 - 3 = \square$
$30 + 7 - 4 = \square$
$30 + 7 - 5 = \square$
$30 + 7 - 6 = \square$
$30 + 7 - 7 = \square$

(iv)
$9 - 9 = \square$
$90 - 90 = \square$
$9000 - 9000 = \square$
$148{,}975 - 148{,}975 = \square$
$\triangle - \triangle = \square$

(b) Division as the Inverse of Multiplication

(i) $12 \times \frac{1}{2} = \square$
$12 \times \frac{1}{3} = \square$
$12 \times \frac{1}{6} = \square$
$12 \times \frac{1}{12} = \square$

(ii) $12 \div 2 = \square$
$12 \div 3 = \square$
$12 \div 6 = \square$
$12 \div 12 = \square$

(iii) $12 \times 6 \div 2 = \square$
$12 \times 6 \div 3 = \square$
$12 \times 6 \div 6 = \square$
$12 \times (6 \div 6) = \square$

(iv) $2 \times 10 \div 10 = \square$
$2 \times 250 \div 250 = \square$
$2 \times \triangle \div \triangle = \square$
$2 \times \frac{10}{10} = \square$
$2 \times \frac{\square}{\square} = \square$

Complementary addition can now be re-examined in the light of inverse operations.

Through past experience, using Cuisenaire rods, the child knows that $17 - 9 = \square$ implies $9 + \square = 17$. Now, through an understanding of inverse operations, the child can say :

If $17 - 9 = \square$, then $9 + 17 - 9 = 9 + \square$ (Equals added to equals)
and $17 + 9 - 9 = \square + 9$ (Commutative law : addition)
and $17 = \square + 9$. (Inverse operation : addition—subtraction)

Using the interrelationships between operations, the child should confidently and meaningfully manipulate equations such as those given below and obtain solutions or otherwise simplify them.

(i) $7 + \square = 12$
$7 + \square - 7 = 12 - 7$
$7 - 7 + \square = 5$
$\square = 5$

(ii) $13\frac{1}{2} - \square = 7$
$13\frac{1}{2} - \square + \square = 7 + \square$
$13\frac{1}{2} = 7 + \square$
$13\frac{1}{2} - 7 = 7 + \square - 7$
$6\frac{1}{2} = \square - 7 + 7$
$6\frac{1}{2} = \square$

(iii) $\square \times 6 = 33$
$\square \times 6 \div 6 = 33 \div 6$
$\square \times 1 = \frac{33}{6}$
$\square = 5\frac{1}{2}$

(iv) $\square \div 7 = 3\frac{1}{7}$
$7 \times \square \div 7 = 7 \times 3\frac{1}{7}$
$\square \times 7 \div 7 = 21 + \frac{7}{7}$
$\square = 22$

(v) $7 + (3 \times \triangle) = 22$
$(3 \times \triangle) + 7 = 22$
$(3 \times \triangle) + 7 - 7 = 22 - 7$
$(3 \times \triangle) = 15$
$3 \times \triangle \times \frac{1}{3} = 15 \times \frac{1}{3}$
$3 \times \frac{1}{3} \times \triangle = 5$
$\triangle = 5$

(vi) $\square \times \frac{3}{4} = 27$
$\square \times \frac{3}{4} \div 3 = 27 \div 3$
$\square \times \frac{1}{4} = 9$
$\square \times \frac{1}{4} \times 4 = 9 \times 4$
$\square = 36$

28

FORMAL PROCESSES

ADDITION

Aim

To extend the child's skill in addition with whole numbers to include thousands.

Notes

1. Teachers should refer to Curriculum Guide, Section F, for the series of activities through which the child has proceeded to reach the final refinement of the algorithm.

2. Section G sees the final refinement of examples in which regrouping of tens and units is necessary. Examples are also extended to include thousands.

3. These extensions are in fact quite small provided the child has mastered addition as presented in Section F.

Development

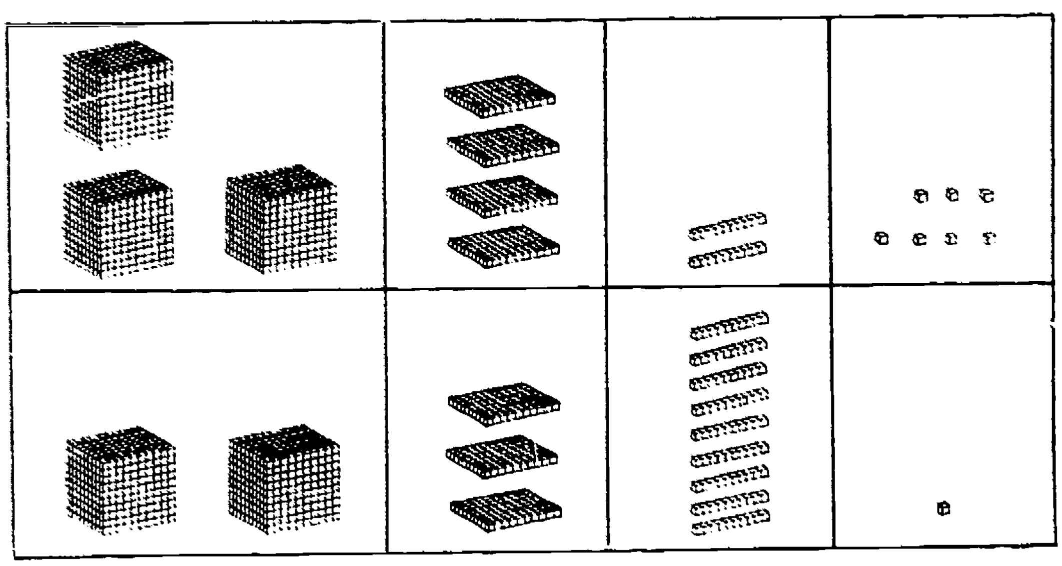
Some initial work might be undertaken with an abacus and a base 10 material such as Dienes's M.A.B. This would reinforce the work completed in Section F. Such material allows the child to combine and regroup collections through physical manipulations.

If the child has not had experience with material such as Dienes's M.A.B. some time will need to be devoted to familiarization. Because the child at this level is more mature than the child at Section B level, the amount of time necessary for this experience will be shorter than that taken with the rods in the earlier section.

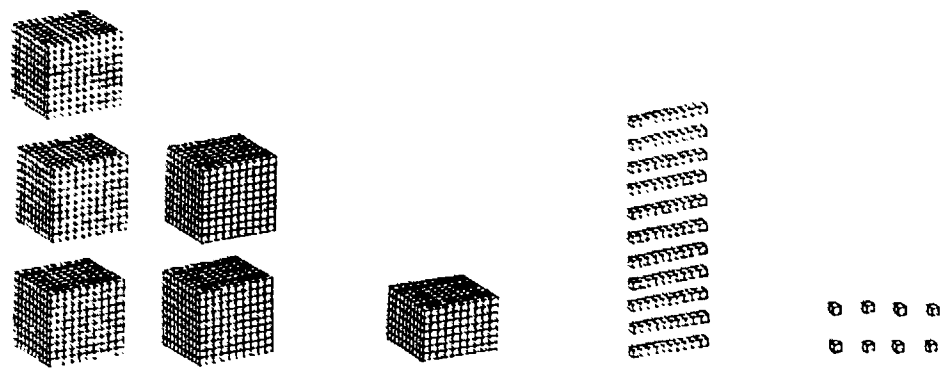
The child may be asked to solve an equation such as—

$$3{,}427 + 2{,}391 = \square.$$

This can be done by using the base 10 material as follows :

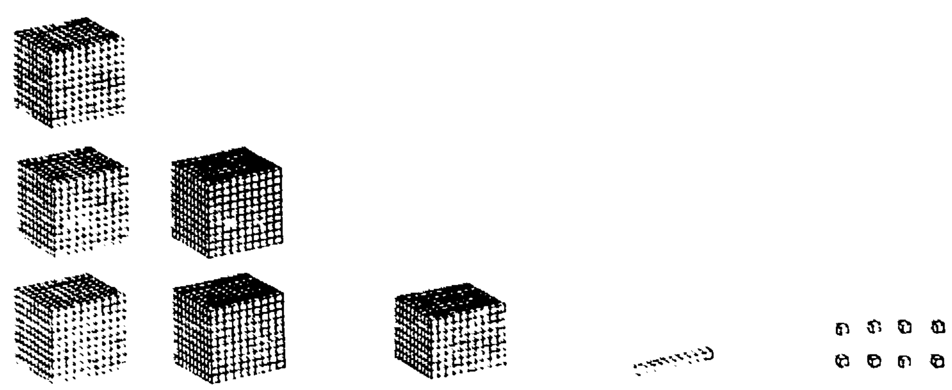


By physically combining the material, the child now has the situation :



5 x 1,000	700	110	8

He is then led to see that the 11 "tens" can be exchanged for 1 "hundred" and 1 "ten". The situation now becomes :

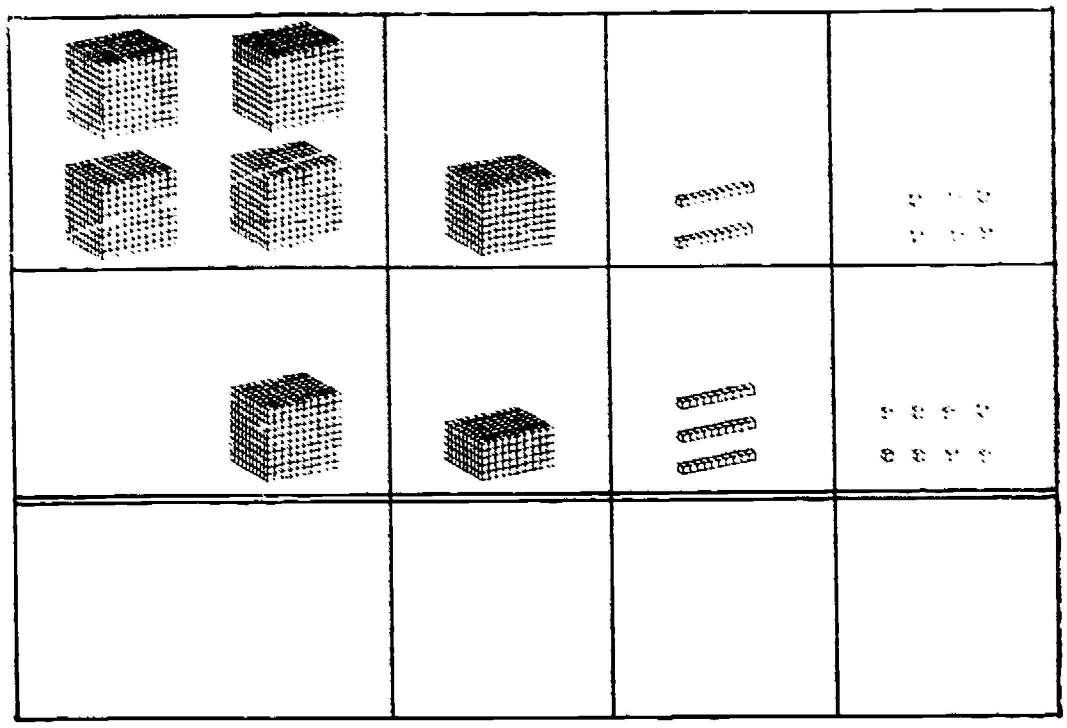


	5 x 1,000	8 x 100	1 x 10	8 x 1
or	5,000	800	10	8
or	5,818			

More complicated situations in terms of regrouping can also be illustrated with the material. After the initial work, the child may be asked to record the steps revealed in the physical manipulations.

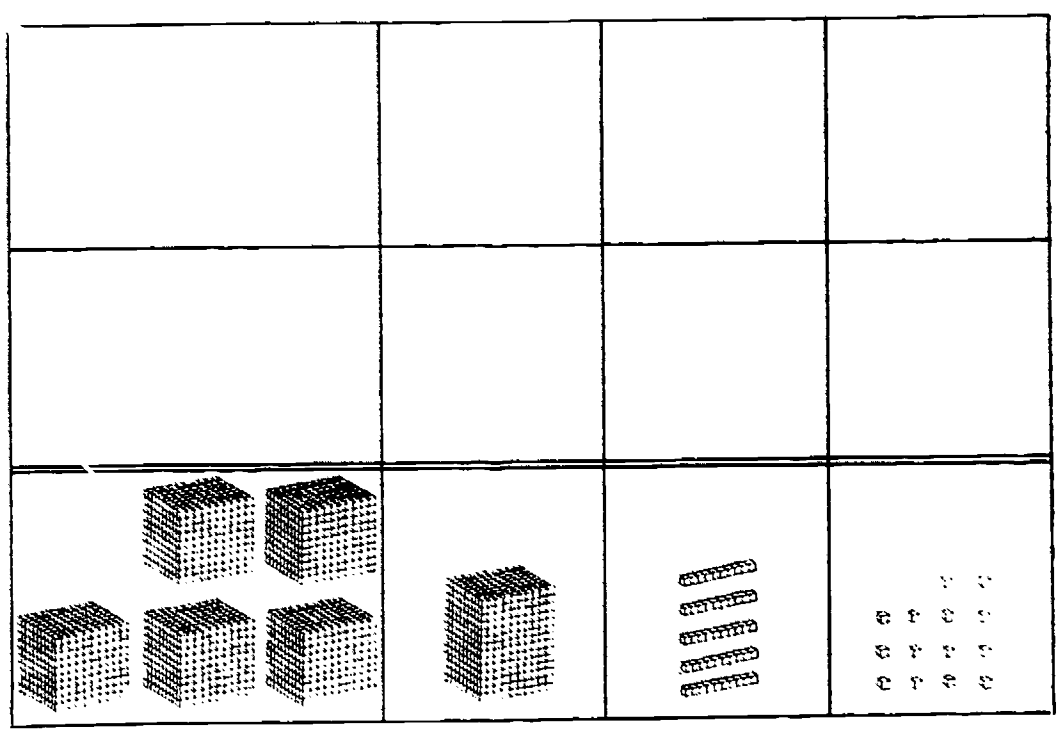
Example : $4{,}926 + 1{,}538 = \square$

30



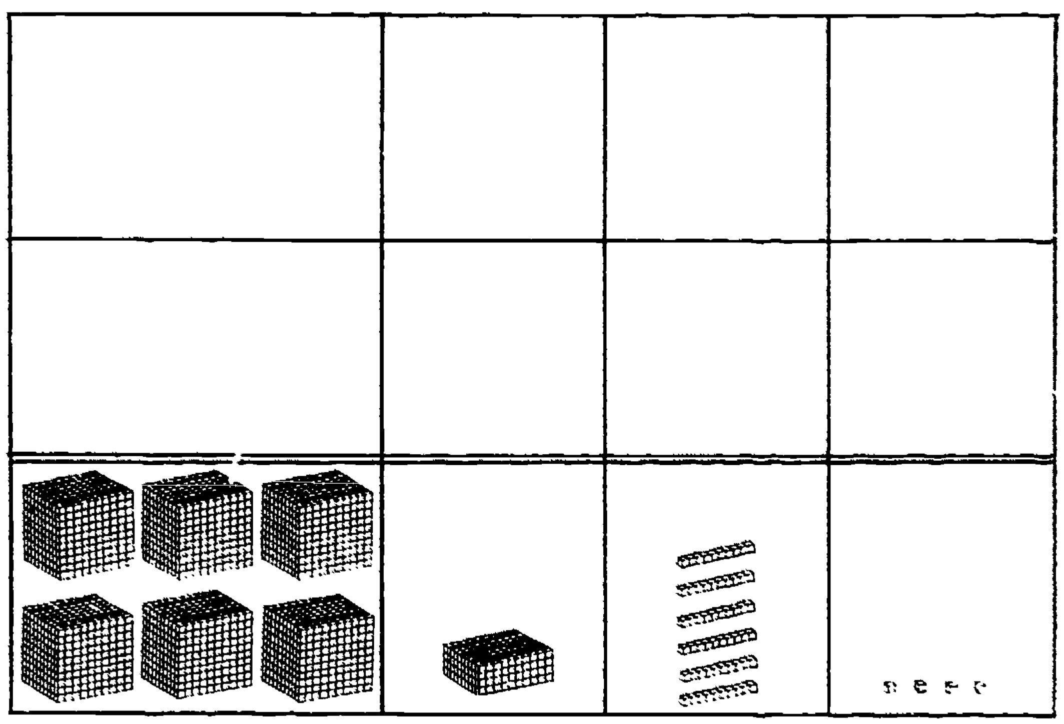
Th	H	T	U
4 1	9 5	2 3	6 8

$$\begin{array}{r} 4{,}000 + 900 + 20 + 6 \\ 1{,}000 + 500 + 30 + 8 \\ \hline \\ \hline \end{array}$$



Th	H	T	U
4	9	2	6
1	5	3	8
5	14	5	14

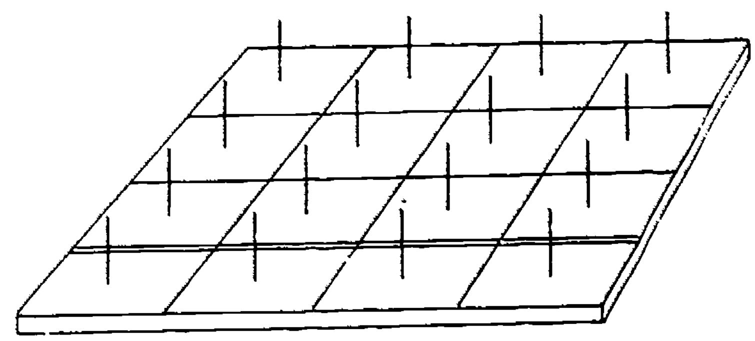
$$
\begin{array}{rrrr}
4{,}000 + & 900 + & 20 + & 6 \\
1{,}000 + & 500 + & 30 + & 8 \\
\hline
5{,}000 + & 1400 + & 50 + & 14 \\
\hline
\end{array}
$$



Th	H	T	U
4	9	2	6
1	5	3	8
5	14	5	14
6	4	6	4

$$
\begin{array}{rrrr}
4{,}000 + & 900 + & 20 + & 6 \\
1{,}000 + & 500 + & 30 + & 8 \\
\hline
5{,}000 + & 1400 + & 50 + & 14 \\
6{,}000 + & 400 + & 60 + & 4 \\
\hline
\end{array}
$$

A development similar to that presented above with the base 10 material could be followed when using an abacus. A spike abacus could be utilized or, perhaps more appropriately, an abacus sometimes referred to as the "Arithmetic or Sealey Abacus". The latter consists of a base into which pieces of dowel or wire, all of the same height, are fixed. Each dowel can accommodate exactly nine beads. Beads of four distinctive colours are required. Note that variations of this model could prove helpful in handling ten-thousands, hundred-thousands, and decimals.

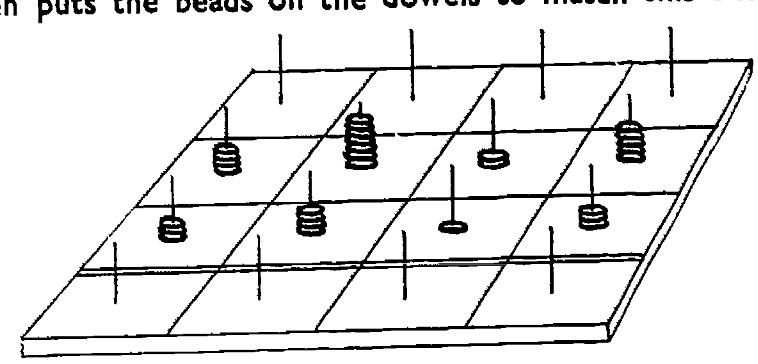


As with the Cuisenaire rods, colour indicates a relationship between the beads. For example, the rules of the game could indicate that 10 green beads are equivalent in value to 1 red bead, 10 red beads are equivalent to 1 blue bead, and 10 blue beads are equivalent to 1 white bead.

The child is given a situation such as :

	white	blue	red	green
	4	7	2	5
+	3	4	1	3

He then puts the beads on the dowels to match this situation.



The child then physically combines all beads of the same colour and places them in the answer row of the abacus. However, in this example, he discovers that he is unable to place all of the blue beads on the piece of dowel in the appropriate place.

To solve this difficulty he converts ten of these blue beads to one white bead, which is then added to the answer dowel in the white column. The final result can then be recorded as :

white	blue	red	green
4	7	2	5
3	4	1	3
8	1	3	8

The child at this level will move quickly from the colour names to the conventional names of thousands, hundreds, tens, and units.

In the development of the four processes with whole numbers, three distinct aspects can be identified.

> *First*, an understanding of the nature of the operations. This has received detailed treatment in previous guides.
>
> *Second*, an understanding of the computational techniques involved. Since the solution to many problems cannot be written down immediately by using a single application of a basic mathematical operation, a written technique is an appropriate aid to reaching a result. Section F Guide provides a development which leads to an efficient algorithm in three of the processes. Section G sees the completion of this development.
>
> *Third*, the development of mastery in using the technique or algorithm. By the completion of Section F, children should have reached a high level of proficiency in addition, subtraction, and multiplication.

An examination of examples involving hundreds, tens, and units has been undertaken in Section F. Now there is an extension to include thousands. The steps given below will reinforce the understanding of the algorithms developed in the previous section. However, their use will not be extensive. Some teachers may require a child at the Section G level to see the same addition exercise in different ways, as shown below.

$$3{,}271 + 2{,}528 = \square$$

3 thousands	2 hundreds	7 tens	1 unit
2 thousands	5 hundreds	2 tens	8 units
5 thousands	7 hundreds	9 tens	9 units

$$\begin{array}{l} 3{,}000 + \ 200 + 70 + 1 \\ 2{,}000 + \ 500 + 20 + 8 \\ \hline 5{,}000 + \ 700 + 90 + 9 \\ \hline \end{array}$$

$$\begin{array}{r} 3{,}271 \\ 2{,}528 \\ \hline 9 \\ 90 \\ 700 \\ 5{,}000 \\ \hline 5{,}799 \\ \hline \end{array}$$

34

$$2{,}934 + 1{,}438 = \square$$

$$\begin{array}{r}
2{,}000 + 900 + 30 + 4 \\
1{,}000 + 400 + 30 + 8 \\
\hline
3{,}000 + 1{,}300 + 60 + 12 \\
\text{or} \qquad\qquad\qquad\qquad \\
4{,}000 + 300 + 70 + 2 \\
\hline
\end{array}$$

This step may be justified as follows :

$3{,}000 + (1{,}000 + 300) + 60 + (10 + 2)$ (Renaming)

$(3{,}000 + 1{,}000) + 300 + (60 + 10) + 2$ (Associative property)

$4{,}000 + 300 + 70 + 2$ (Renaming)

$4{,}372$

$$\begin{array}{r}
2{,}934 \\
1{,}438 \\
\hline
12 \\
60 \\
1{,}300 \\
3{,}000 \\
\hline
4{,}372 \\
\hline
\end{array}$$

Types of Examples

Computational examples will include the following types and combinations of these :

Addition :

$$\begin{array}{r}
3{,}241 \\
1{,}652 \\
\hline
\\
\hline
\end{array}
\qquad
\begin{array}{r}
1{,}421 \\
1{,}347 \\
5{,}211 \\
\hline
\\
\hline
\end{array}
\qquad
\begin{array}{r}
1{,}214 \\
3{,}122 \\
2{,}531 \\
3{,}112 \\
\hline
\\
\hline
\end{array}$$

(No regrouping)

$$\begin{array}{r}
3{,}246 \\
1{,}315 \\
4{,}127 \\
\hline
\\
\hline
*
\end{array}
\qquad
\begin{array}{r}
4{,}851 \\
2{,}712 \\
1{,}934 \\
\hline
\\
\hline
*
\end{array}
\qquad
\begin{array}{r}
3{,}247 \\
1{,}498 \\
2{,}634 \\
\hline
\\
\hline
***
\end{array}$$

(Regrouping)

5,032	1,403	2,009	(Zeros in addends)
1,450	2,740	7,008	
3,217	4,519		
	1,024		

3,436	2,324	3,278	(Zeros in total)
1,192	1,452	1,167	
4,181	1,315	1,555	
	3,913		

1,437	3,517	8	(Differing numbers of digits in the addends)
51	154	4,008	
211	6	319	
	4,315	24	

Variations in Presentation

When giving children computational examples to solve, the following variations in presentation could be used. *(These, with adaptations, will apply to all of the four processes with whole numbers.)*

986	$284 \times 9 = \square$
37	
214	$\square \times 7 = 3{,}724$
500	
	$635 = \square - 271$

Replace each ⊙ with <, >, or = to make each of the following statements true :

$486 \times 37 \odot 17{,}980$

$489 + 32 + 214 \odot 4{,}599 - 3{,}864$

Which of the following statements are true ?

$684 \div 4 = 900 - 749$

$274 \times 9 = 598 + 47 + 1{,}106 + 415$

$9{,}502 - 709 = 3 \times 977$

Which of the following statements are false ?

$344 \div 7 > 148 \div 3$

$374 + 1{,}903 + 6{,}837 < 2{,}004 + 3{,}147 + 4{,}802$

$9{,}040 - 2{,}731 = 7{,}043 - 734$

Which of the following answers would make the open sentences true ?

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Open Sentence	Answers
173 x 25 > □	A. 4,425 B. 4,320 C. 3,245 D. 4,325
874 ÷ 9 < □	A. 95 B. 97 C. 100 D. 96

Find the total of :

3 tens 9 thousands 4 units, and 2 thousands 5 hundreds 6 units 8 tens.

Find the difference between :

8 tens 5 units 6 thousands, and 2 thousands 5 hundreds.

SUBTRACTION

Aim

To extend the child's skill in subtraction with whole numbers to include thousands.

Notes

1. It is important to reinforce work that has been undertaken in earlier sections. Use of a structured aid, such as base ten material, emphasizes place value. At the same time it introduces additional variations in visual pattern from those suggested in activities given in Section F of the Curriculum Guide.

2. Knowledge and understanding of the various views taken of the operation of subtraction as given in Section F of the Curriculum Guide (page 94) should be maintained. These were—

(*a*) the difference between two numbers ;

(*b*) complementary addition ;

(*c*) " take away ".

Development

1. Initially the child might be asked to work examples such as 741 − 482 through the use of structured materials. The child could approach the example in a number of ways.

(*a*) He might construct two towers with base ten material—one representing 741 and the other 482—and then find the amount by which the larger tower exceeds the smaller.

(*b*) Alternatively, the child might adopt an approach that could be classified as complementary addition. Here the child adds pieces to the shorter tower until it is equal to the other tower. The value of the material added is then found.

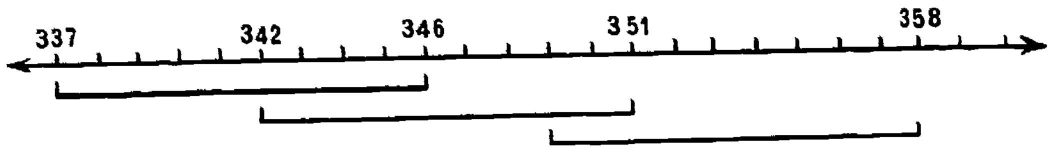
(*c*) A third approach would be to construct only the tower representing 741 and then to take away pieces representing 482. The value of the remaining pieces then gives the child his answer.

Teachers should encourage the child to use a variety of methods.

2. At Section G level, the aim is for the child to understand and have mastery of an algorithm for subtraction. This mastery should extend to examples with minuends up to and including 9,999. The actual technique used in the algorithm may vary from child to child. Generally, a child's preference will depend upon his past successes and the influence of the teacher in directing him towards one particular approach. Preference has been given to "equal additions" in previous guides. This has been continued with the development given below.

From earlier work the child knows that when equal numbers are added to unequal numbers there is no change in the difference. This can be revised very simply by using a number line.

Example : 346–337 = ☐



$$346 - 337 = 9$$
$$(346 + 5) - (337 + 5) = 9$$
$$(346 + 12) - (337 + 12) = 9$$

In Section F, the child proceeded to the final refinement in examples involving two digits where no digit in the subtrahend exceeded in value the corresponding digit in the minuend.

For example : 84 − 73 = ☐

An extension to 3 digits in examples of this type presents few added complications. A suitable sequence from this point is as follows :

$$453 - 139 = \square$$

(i)

$$\begin{array}{l} 400 + 50 + \overset{13}{\cancel{3}} \\ 100 + \overset{40}{\cancel{30}} + 9 \\ \hline 300 + 10 + 4 = 314 \\ \hline \end{array}$$

(ii)

$$\begin{array}{ccc} 4 & 5 & \overset{13}{\cancel{3}} \\ 1 & \overset{4}{\cancel{3}} & 9 \\ \hline 3 & 1 & 4 \\ \hline \end{array}$$

(iii)

$$\begin{array}{r} 453 \\ 139 \\ \hline 314 \\ \hline \end{array}$$

In each of these steps the ten has been added to both the minuend and the subtrahend. The child should also be aware that in some cases it may be to his advantage to add a different number.

For example : 621 − 294 = ☐

Here the child may add the number six to both the minuend and the subtrahend. The situation then becomes 627 − 300.

An alternative technique that some children may use is based upon their experiences in renaming numbers. This is the decomposition method.

For example : 528 − 175 = ☐

(i)
$$\begin{array}{lllll} \overset{4}{\cancel{5}} \text{ hundreds} & + & \overset{12}{\cancel{2}} \text{ tens} & + & 8 \text{ units} \\ 1 \text{ hundred} & + & 7 \text{ tens} & + & 5 \text{ units} \\ \hline 3 \text{ hundreds} & + & 5 \text{ tens} & + & 3 \text{ units} \\ \hline \end{array}$$

(ii)
$$\begin{array}{lllll} \overset{400}{\cancel{500}} & + & \overset{120}{\cancel{20}} & + & 8 \\ 100 & + & 70 & + & 5 \\ \hline 300 & + & 50 & + & 3 \\ \hline \end{array}$$

(iii)
$$\begin{array}{ccc} 5 & 2 & 8 \\ 1 & 7 & 5 \\ \hline 3 & 5 & 3 \\ \hline \end{array}$$

By the end of Section G it is most likely that the child will automatically adopt one computational procedure. He will apply this in a rote fashion, giving little thought to the reasons why he can operate in such a fashion. However, if queried about the manipulations he carries out, he should be able to justify them. Moreover, if asked, he should be able to use other computational methods. The ability of a child to use a variety of computational procedures and to select the method most appropriate to a given situation may be a guide to his level of understanding.

Types of Examples

Computational examples should include the following types.

741 − 213 ——— ***	6,892 − 1,327 ——— *	837 − 142 ——— *	7,324 − 5,193 ——— *	3,278 − 1,743 ——— *	(Digits in subtrahend of greater value than corresponding digits in minuend)
624 − 478 ——— **	4,842 − 1,157 ——— **	2,572 − 1,934 ——— * *	5,136 − 3,897 ——— ***		

256 − 102	3,948 − 2,005		(Zeros in subtrahend)
640 − 217	7,034 − 1,513	6,000 − 3,471	(Zeros in minuend)
5,274 − 3,173	3,738 − 1,736		(Zeros in solution)
7,394 − 173	9,064 − 1,391	6,802 − 514	(Combinations)

Variations in Presentation

For variation in presentation of practice examples refer to "Addition ", pages 36-37.

MULTIPLICATION

Aim

To extend the child's skill in multiplication with whole numbers, with multiplicands to 999, multipliers to 99.

Notes

1. It is expected that the child will have mastered the following prerequisites :

(*a*) Understanding of the operation of multiplication ;

(*b*) knowledge of basic multiplication and addition facts ;

(*c*) place value ;

(*d*) commutative and associative properties of multiplication ;

(*e*) distributive property of multiplication over addition.

2. There is a need for the child to develop a computational technique for handling large numbers.

Finding the product of two one-digit factors becomes a mental operation once automatic response has been achieved in basic number combinations. However, when a number of two or more digits is

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multiplied by a one-digit number from 2 to 9, for example 23 x 6, the products of such factors are usually beyond the range of basic number combinations in which the child is expected to have achieved automatic response. It would be unrealistic and unreasonable to expect the child to memorize such combinations.

A process by which problems involving large numbers can be solved is necessary. Such a process can be established using basic multiplication combinations, decade addition combinations, and knowledge of the basic properties of multiplication.

The solution to the equation $49 \times 17 = \square$ will not be produced automatically. Some children may produce the solution after much mental effort, but some written technique is now essential to record partial products and their sums. $49 \times 17 = 833$ is a statement that expresses in mathematical language an operation in mathematics. To record exactly what was done to solve this problem we may write as follows :

$$49 \times 17 = (49 \times 10) + (49 \times 7) = 490 + 343 = 833 .$$

The traditional process is a refinement of the above and is merely the scrap-paper work needed to carry out the operation.

$$\begin{array}{r} 49 \\ 17 \\ \hline 490 \\ 343 \\ \hline 833 \\ \hline \end{array}$$

It is essential that the child should be able to relate the equation form to the traditional form and vice versa.

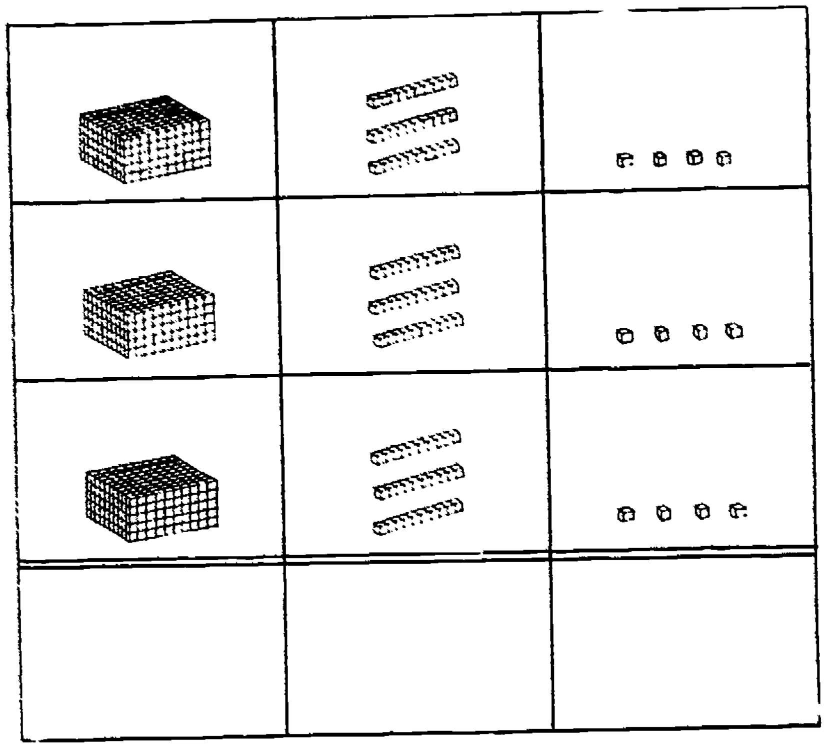
3. The establishment of a process for handling large numbers was commenced in Section F. Section G sees the extension of this work to mastery of the final refinement in examples involving regrouping. Multiplicands to 999 and multipliers to 99 are the upper limits of this section.

Development

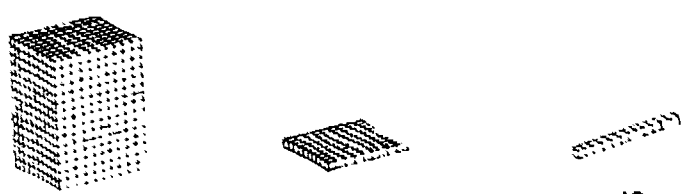
The link between the operations of addition and multiplication, when using larger numbers, can be reinforced by using a base 10 material and an abacus.

For example : 534 x 3

In the illustration below the child is reading 534 x 3 as 534 multiplied by three, that is, three sets of five hundred and thirty-four. See Curriculum Guide, Section F (page 89) for a discussion on the alternative interpretations of the sign " X ".

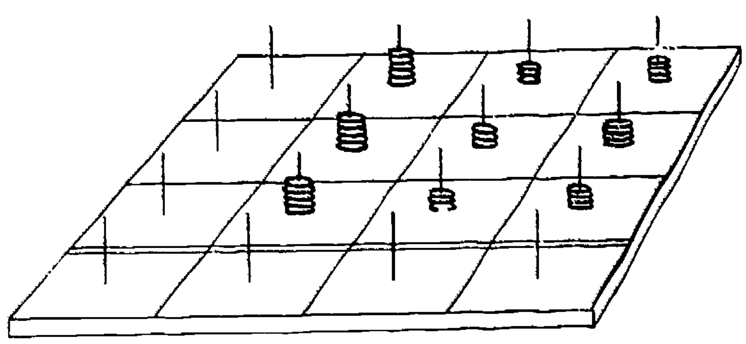


After combining the material the child discovers that he has the following situation :



He then regroups the material to find the standard answer of 1,602.

The "Arithmetic Abacus" as described in Addition (see page 32) can also be used with this topic. The example used above can be illustrated on the abacus as follows :



The process of combining and regrouping the beads is similar to that used with the base 10 material.

These concrete situations can provide a means of illustrating the operation of multiplication as repeated addition. From this and other examples the child can proceed with written work as follows :

$$
\begin{array}{llllll}
(a) & 534 \times 3 & = (500 \times 3) & + (30 \times 3) & + (4 \times 3) \\
 & & = 1{,}500 & + 90 & + 12 \\
 & & = 1{,}590 + 12 & & \\
 & & = 1{,}602 & &
\end{array}
$$

$$
\begin{array}{lrcr}
(b) & 500 + 30 + 4 & & 1{,}500 \\
 & \times 3 & & 90 \\
 & & & 12 \\
\hline
 & 1{,}500 + 90 + 12 & = & 1{,}602 \\
\hline
\end{array}
$$

$$
\begin{array}{lrl}
(c) & 534 & \\
 & \times 3 & \\
\hline
 & 12 & (4 \times 3) \\
 & 90 & (30 \times 3) \\
 & 1{,}500 & (500 \times 3) \\
\hline
 & 1{,}602 & \\
\hline
\end{array}
$$

$$
\begin{array}{lr}
(d) & 534 \\
 & \times 3 \\
\hline
 & 1{,}602 \\
\hline
\end{array}
$$

Later in the section when the child is ready to proceed to examples involving multipliers larger than twelve, concrete illustrations should be unnecessary. However, the child's understanding of the algorithm should be extended by using a sequence such as the following, with carefully graded examples.

Stages of refinement

Consider 35 x 42 = $\square$

$$
\begin{array}{lrl}
(a) & 35 & \\
 & 40 + 2 & \\
\hline
 & 10 & (2 \times 5) \\
 & 60 & (2 \times 30) \\
 & 200 & (40 \times 5) \\
 & 1{,}200 & (40 \times 30) \\
\hline
 & 1{,}470 & \\
\hline
\end{array}
\qquad\qquad
\begin{array}{lr}
(b) & 35 \\
 & \times 42 \\
\hline
 & 10 \\
 & 60 \\
 & 200 \\
 & 1{,}200 \\
\hline
 & 1{,}470 \\
\hline
\end{array}
$$

$$\begin{array}{r} \text{(c)}\ \ 35 \\ \times\ 42 \\ \hline 70 \\ 1{,}400 \\ \hline 1{,}470 \\ \hline \end{array}$$

Types of Examples

When selecting computational examples, consideration should be given to the following types:

$$\begin{array}{r} 234 \\ \times\ 2 \\ \hline 468 \\ \hline \end{array}$$ (No regrouping)

$$\begin{array}{r} 127 \\ \times\ 3 \\ \hline 381 \\ \hline * \end{array} \qquad \begin{array}{r} 317 \\ \times\ 4 \\ \hline 1{,}268 \\ \hline *\ * \end{array} \qquad \begin{array}{r} 732 \\ \times\ 8 \\ \hline 5{,}856 \\ \hline *** \end{array}$$ (Regrouping)

$$\begin{array}{r} 261 \\ \times\ 5 \\ \hline 1{,}305 \\ \hline \end{array} \qquad \begin{array}{r} 146 \\ \times\ 5 \\ \hline 730 \\ \hline \end{array} \qquad \begin{array}{r} 625 \\ \times\ 12 \\ \hline 7{,}500 \\ \hline \end{array}$$ (Zeros in products)

$$\begin{array}{r} 450 \\ \times\ 6 \\ \hline 2{,}700 \\ \hline \end{array} \qquad \begin{array}{r} 906 \\ \times\ 7 \\ \hline 6.342 \\ \hline \end{array}$$ (Zeros in multiplicand)

Similar points should be considered when selecting examples involving multipliers larger than twelve.

Variations in Presentation

For variations in presentation of examples refer to "Addition" (pages 36–37).

DIVISION

Aim

To develop the child's skill in division with whole numbers and to extend his understanding of the operation with dividends to 9,999 and divisors to 12.

Notes

1. In order to understand the actual techniques for multiplying and dividing numbers, it is essential that children understand the nature and the properties of multiplication and division as operations of mathematics.

In earlier sections, multiplication for whole numbers was considered as a special case of addition where all the addends are the same number. Likewise, division was considered as an operation suggested by partitioning a collection into smaller, equivalent collections.

In Section F, the relationship between multiplication and division was considered. Division was considered in terms of multiplication ; that is, it is an operation between two whole numbers (a product and a factor of the product) determining an unknown factor.

Factor x Factor = Product.

Product ÷ Factor = Factor.

If the child knows the basic multiplication combinations and the relation between multiplication and division, and if his response to multiplication facts is automatic, then the need for independent division facts is reduced.

2. Since the solution to many problems cannot be written down at once by using a single application of a basic mathematical operation, a written technique is an essential aid to reaching a result.

3. One computational process for division is based on successive subtractions involving multiplication. Estimates are involved in all but the simplest examples. It should be remembered that the long division process in common use today is a short cut that has been developed by mathematicians over a long period of time. In the hands of a skilled calculator it is a time-saving device, but it is too difficult to be used in the initial development of the written process. Many of the difficulties in teaching long division have arisen in the past because the traditional form was used as the only teaching process. Generally, in an attempt to assist the child in his mastery of the process, he was given a series of rules to apply. An example of this was the sequence : Divide, multiply, subtract, compare, bring down. Now the child is encouraged to understand the process by proceeding through a sequence of refinement which leads to the traditional algorithm. This means that many of the old difficulties faced by the child are overcome. However, it is most important that the child should have achieved reasonable skill in subtraction and multiplication processes and in estimation before he proceeds very far. This is necessary if success in using this process is to be achieved.

It is important that each child be permitted to proceed through the sequence of refinement at a pace appropriate his growing skill and understanding.

Development

1. *Division as Repeated Subtraction*

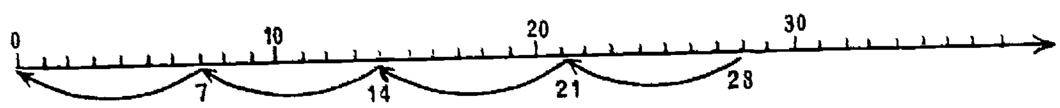
This link is not new to the child since he has examined it in earlier sections. It could be revised through activities such as :

(*a*) "John has a box containing 54 counters. He wishes to put all of these counters into bags, each bag containing 3 counters. How many bags will he need ?"

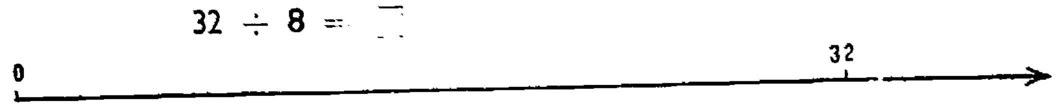
The child can set up this practical problem and then proceed to remove counters from the box and form collections, each of which contains three counters. When he has completed this physical subtraction he can count the number of collections.

(*b*) Use the number lines provided to solve each equation below :

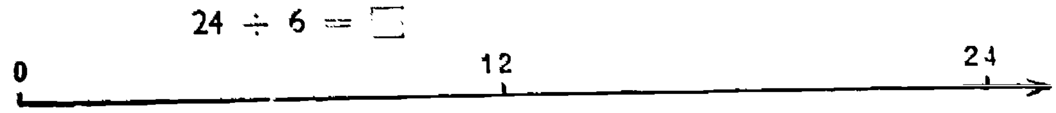
$28 \div 7 = \square$



$32 \div 8 = \square$



$24 \div 6 = \square$



(c) Solve the following equations :

$42 - \square - \square - \square - \square - \square - \square - \square = 0$ $\quad$ $27 - \triangle - \triangle - \triangle = 0$

$42 - (\square \times 7) = 0$ $\quad$ $27 - (\triangle \times 3) = 0$

$42 \div 7 = \square$ $\quad$ $27 \div 3 = \triangle$

2. *Division as the Inverse of Multiplication*

Activities such as the following could be used to reinforce the child's understanding of the relationship between division and multiplication.

(*a*) Think of a number.

Multiply it by 7.
Divide by 7.
What is the result ?
How does the result compare with the number you first thought of ?

A number of similar examples may assist the child to appreciate the "doing and undoing" relationship that exists between multiplication and division.

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(*b*) Solve each equation :

$12 \times 2 \div 2 = \square$
$54 \div 6 \times 6 = \square$

Can you write the answer to each of the following equations without making any calculations ?

$144 \div 12 \times 12 = \square$
$127 \times 2 \div 2 = \square$
$156 \div 3 \times 3 = \square$

(*c*) Find the missing numerals :

Factor ..		7		9	6
Factor ..	4	8	8	12	
Product ..	24		96		54

Such an activity aims to strengthen the child's understanding of division as finding the unknown factor when the product and another factor are known.

(*d*) For each multiplication statement below write a corresponding division statement :

Multiplication	*Division*
$9 \times 3 = 27$	$27 \div 3 =$
$12 \times 4 = 48$	$\_ \div \_ = \_$
$36 \times 2 =$	$\_ \div \_ = \_$

3. *Estimation in Division*

An essential part of the topic "Division with Whole Numbers" is the development of the child's skill in estimating quotients. The more efficient he becomes in making these estimates, the closer he will come to the traditional algorithm for long division. Also, efficiency reduces the possibility of computational slips.

The following are suggested activities :

(*a*)

$6 \div 6 = 1$	$28 \div 7 = 4$	$28 \div 7 = 4$
$60 \div 6 = \square$	$280 \div 7 = \square$	$280 \div 70 = \square$
$600 \div 6 = \square$	$2{,}800 \div 7 = \square$	$2{,}800 \div 700 = \square$

$4 \times 7 = 28$	$4 \times 7 = 28$
$4 \times 70 = \square$	$40 \times 7 = \square$
$4 \times 700 = \square$	$400 \times 7 = \square$

(*b*) How many 5s in 50 ?
How many 5s in 500 ?
How many 5s in 497 ?

(c) Later, children may be grouped in pairs for the following activity. A division problem, written on a card with the correct answer on the reverse side, is given to each group. Each child makes an estimate of the quotient and then checks with the correct answer. The child with the estimate nearest to, but not over, the correct answer scores a point. At the end of the activity the child with the most points wins the game.

(d) The child can use his knowledge of the relationship between division and multiplication. Before dividing in an example such as $346 \div 7$, he can re-phrase the problem in terms of multiplication.

"How many 7s are there in 346 ?"

or

$\square \times 7 \leq 346$ ($\leq$ means "is less than or equal to")

$58 \div 8 = \triangle$	$\square \times 8 \leq 58$
$38 \div 4 = \triangle$	$\square \times 4 \leq 38$
$274 \div 9 = \triangle$	$\square \times 9 \leq 274$

In the above examples, $\square$ stands for the largest whole number for which the sentence is true.

4. *The Division Algorithm*

In the suggested approach, children are introduced to the long division form at the outset. The process and stages of refinement that will take place as the child matures and becomes more skilled are illustrated in the following examples :

(*a*)

6) 414	
60	10
354	
120	20
234	
180	30
54	
54	9
0	69

(*b*)

6) 414	
180	30
234	
180	30
54	
54	9
0	69

(*c*)

6) 414	
360	60
54	
54	9
0	69

To facilitate the division procedure illustrated above, the partial quotients (estimates) are written to the right of a vertical line as illustrated. The complete quotient is the sum of the partial quotients or estimates.

The three examples of the process illustrate stages of refinement. The first shows the least mature approach, the second shows an increase in skill of estimating partial quotients, and the third illustrates the most efficient and mature approach.

The child should be led to discover that in selecting a partial quotient it is better to under-estimate than to over-estimate.

For example :

524 ÷ 9

9	524	
	360	40
	164	
	180	20

Twenty times nine is too great, so the child must erase this section of his work and go back and seek a more suitable quotient.

Eventually the child should reach the decision to *round the dividend down*, that is, to use the best under-estimate. By applying such a generalization the child will not over-estimate.

567 ÷ 7

7	567		How many 7s in 56 tens ?
	560	80	Eighty.
	7		
	7	1	
	0	81	

295 ÷ 9

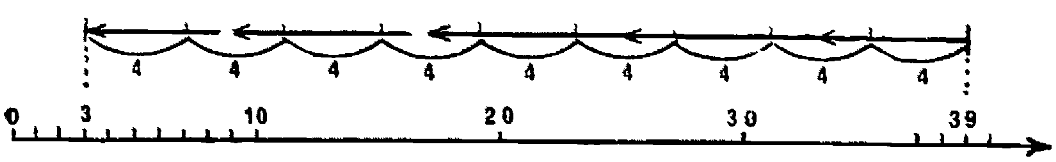
9	295		How many 9s in 29 tens ?
	270	30	
	25		How many 9s in 25 ?
	18	2	
	7	32	

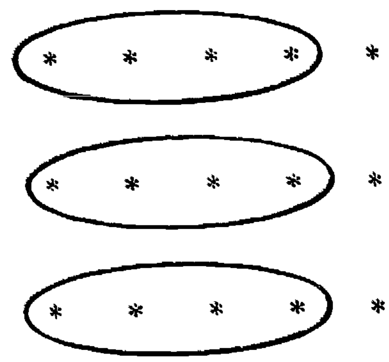
5. *Division with Remainders*

The initial work in this section has been concerned with exact division. However, as the child's understanding of, and skill with, the process grows, he should be introduced to the idea of remainders.

Activities.

(*a*) 39 ÷ 4 = □



(b)  How many sets of 4 can be found in this array ?

(c) Present the child with a set of 23 counters. Ask him to remove 3 at a time.
"How many sets of three were you able to make ?"

(d) "Mark has 34 one-cent coins. How many five-cent coins can he get for them ?"

Notation

Introductory activities such as the above will lead quickly into vertical division notation.

5) 34	
5	1
29	
5	1
24	
5	1
[illegible]	
5	1
14	
5	1
9	
5	1
4	6

5) 34	
30	6
4	6

Writing Answers

Often it has been the practice to ask the child to go one step further than that presented above and write his answer in the following form—

Answer : 6, remainder 4 *or* 6, r. 4

While in normal computation this method of recording is preferred, the child should be aware of the significance of this remainder. The

interpretation given to a remainder depends upon the context out of which it arose. Three examples are given below :

(i) "A station-wagon is used to carry twenty-seven children to a football match. There is room for seven children on each trip. How many trips are necessary ?"

In this situation, after the computational procedure has been completed, the remainder indicates the need for an additional trip. The correct answer to the problem is 4.

(ii) "Six children are to receive equal shares from 83 lollies. What is the largest number of lollies each child can receive ?"

In this case each child will receive 13 lollies, the remaining five are not distributed. Here the remainder is dropped from the final answer.

(iii) "A rope 94 feet long is cut into 5 equal pieces. How long is each piece ?"

An answer of 18, r. 4 has no meaning in such a context. The remainder should be expressed as a fraction. $18\frac{4}{5}$ ft.

(Refer to Curriculum Guide, Section F, p. 101.)

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FRACTIONS

AIM

To extend the study of fractions in terms of the following aspects :

A. A fraction as a number, as a remainder in division, and as a notation for division.

B. Addition and subtraction of vulgar fractions.

C. Multiplication of vulgar fractions.

D. Addition and subtraction of decimal fractions.

E. Multiplication of decimal fractions.

NOTES

The specific aims of the various subsections (A–E above) are set out at the beginning of each subsection.

The child has developed considerable understanding of, and skill with, fractions in preceding sections. It is upon this basis that this section builds. See Curriculum Guide, Section F, pages 68–80.

In this extension, use of concrete aids will still be necessary for the majority of children, but the extent of this use will vary with different aspects of the topic. Rods, area diagrams, number lines, and separate objects can be utilized. Generally, these aids will provide a means of introducing work and aiding transfer to more abstract activities with numerals. It is most important that children see the basic ideas in a variety of visual patterns. The following material indicates suitable activities, not necessarily in the order in which they would be presented to children.

A. A FRACTION AS A NUMBER, A REMAINDER, AND A NOTATION

Aim

To extend the understanding of, and skill with, fractions as :

Numbers greater than 1 ;
whole numbers ,
remainders in division ;
a variation in notation for division.

Whole Numbers and Numbers Greater Than One

More explicit activities should be undertaken at this stage so that the child knows that a fraction can be less than, greater than, or equal to one.

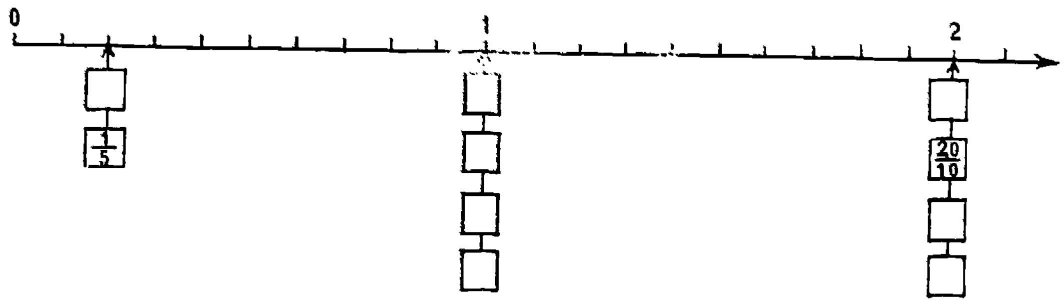
He should have the ability to express fractions that are larger than one in "mixed form" and be aware that a number written in such a way involves addition. For example, $2\frac{1}{2} = 2 + \frac{1}{2}$. "Mixed form" is a more precise term for "mixed number".

Activities

1. Number Lines

Use of number lines as suggested in Section F, page 74, continues to form an important part of the work in this section. An exercise such as the following could also be used :

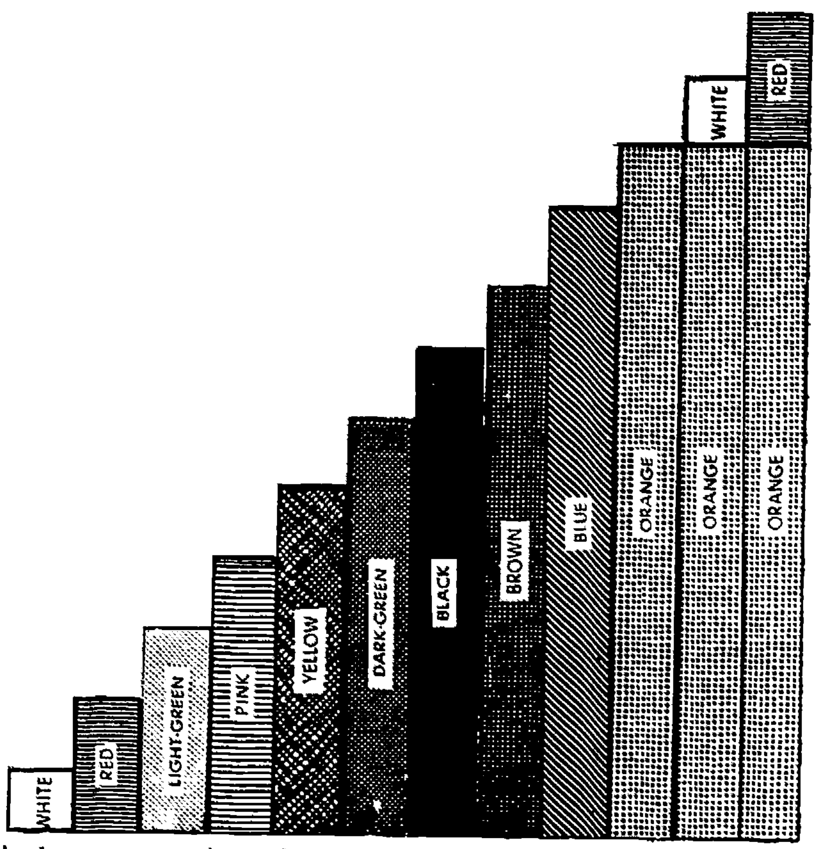
Give different fractional names for each of the points marked on the number line below.



2. Rods

Activities with a staircase of rods continue to be useful.

Consider this staircase :



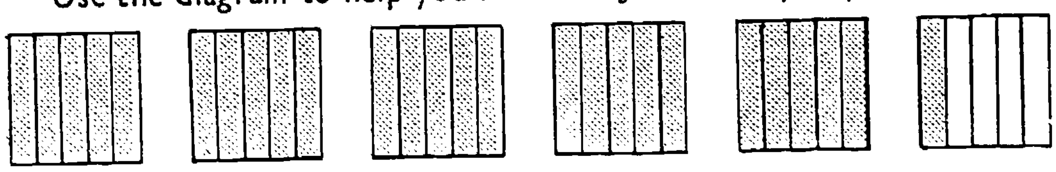
The dark-green rod is the unit of measure.

Name the number value of each rod.
Which rods have values less than one ?
Which rods have values greater than one ?
State the value of each rod greater than one in mixed form.
Which fraction in the sequence comes immediately before one ?
Which fraction in the sequence comes immediately after one ?
Which is greater, 1 or $\frac{7}{6}$? By how much ?

A similar procedure can be taken using other rods as the unit measure, for example, light-green or pink.

3. *Area Diagrams*

Use the diagram to help you rename $5\frac{1}{5}$ in as many ways as possible.



For example : $2 + 3 \ + \frac{1}{5}$

$6 - \frac{4}{5}$

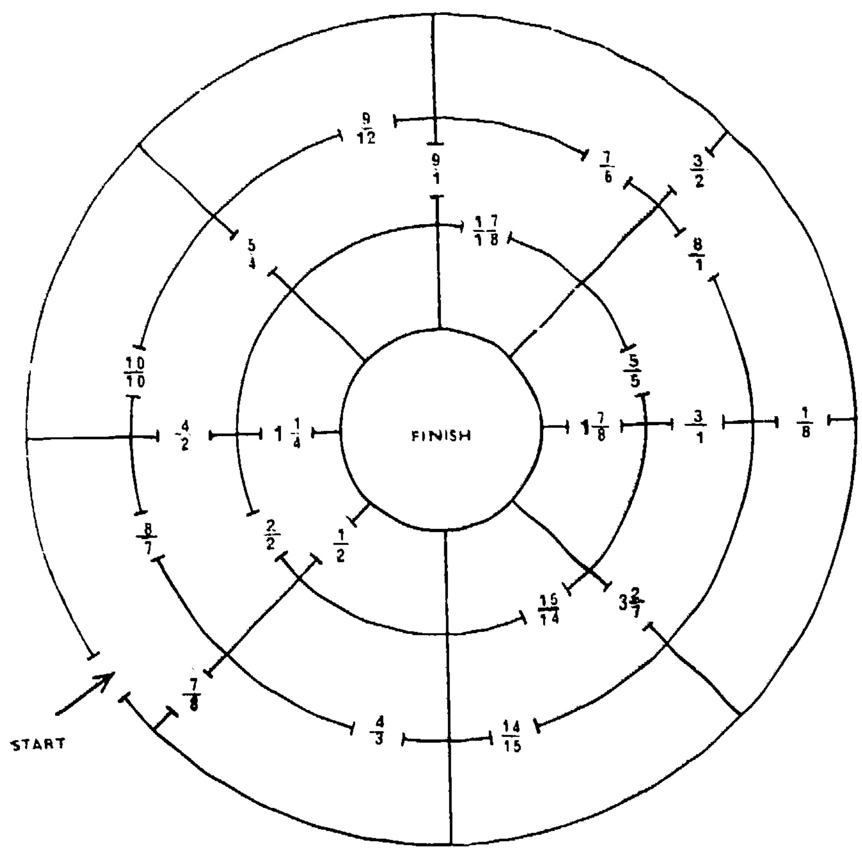
$5 \times 1 \ + \frac{1}{5}$

$(5 \times \frac{5}{5}) + \frac{1}{5}$

4. *Mazes*

The basic maze pattern given below can be used with variations. The complexity of the maze, the instructions to the child, and the numerals used can be varied according to the purpose of the activity and the ability of the child.

For example : "Use the numerals to find your way through the maze. Always take the pathway indicated by a fraction larger than one."



5. *Card Games*

Card games can also be used to reinforce the child's knowledge of a fraction as a number.

One such game, modelled on "Strip Jack Naked", is presented here.

Two to four children can play this game. The pack of cards should be made up of three different sets of fractions—numbers less than one, numbers greater than one, and numbers equal to one. A suitable pack consists of—

32 cards showing fractions less than one,
12 cards showing fractions greater than one, and
4 cards showing fractions equal to one.

To play the game the cards are shared equally among the players. Each player in turn throws out a card face up. When a card showing a numeral greater than, or equal to, one is tabled, the next player must pay a penalty. If the numeral is equal to one the payment is one card. If it is greater than one the penalty is three cards. All cards on the table become the property of the player who originally played the penalty card. However, if in making the payment a player also tables a penalty card, the contract to pay is cancelled and the next player must meet the obligation of the newly played card. The winner is the person with the most cards at the conclusion of the game.

6. *Numerical Activities*

Example 1

Supply the missing signs :

$$4\tfrac{2}{3} = 4 \bigtriangleup \tfrac{2}{3}$$
$$= 2 \bigtriangleup 2 \bigtriangleup \tfrac{2}{3}$$
$$= 3 \bigtriangleup \tfrac{3}{3} \bigtriangleup \tfrac{2}{3}$$

Example 2

Circle the fractions that are greater than one :

$\frac{5}{8}$, $\frac{7}{1}$, $\frac{5}{9}$, $\frac{1}{1}$, $\frac{4}{3}$, $\frac{7}{5}$, $\frac{50}{50}$.

Example 3

Replace each $\odot$ with $>$, $<$, or $=$ to make each sentence true :

$\frac{2}{3}$ of a yard $\odot$ $\frac{2}{5}$ of a yard .
$\frac{1}{8}$ of a gallon $\odot$ $\frac{1}{2}$ of a quart.

Example 4

Place each fraction in the appropriate section of the table :

$\frac{5}{4}$, $\frac{4}{4}$, $\frac{1}{4}$, $\frac{4}{1}$, $\frac{8}{2}$, $\frac{1}{2}$.

Greater than 1	*Equal to 1*	*Less than 1*

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A Fraction as a Remainder

A discussion of a fraction as a remainder is presented on page 52.

Variation in Notation

The child's attention should be drawn to the facts that—

(i) division can be expressed by fraction notation, for example, $27 \div 9 = \frac{27}{9}$;

(ii) in typewritten material it is often necessary to present fractional numbers in the form 1/2 rather than $\frac{1}{2}$.

B. ADDITION AND SUBTRACTION OF VULGAR FRACTIONS

Aim

To extend the study of fractions in regard to:

- The nature of the operations of addition and subtraction in relation to fractions;
- the relationships between the operations of addition and subtraction extended to fractions;
- the development of skill in adding and subtracting fractions with—
  - denominators up to and including sixteenths, but excluding elevenths and thirteenths;
  - answers not necessarily expressed in lowest terms;
  - unlike but related fractions using up to three addends;
  - examples involving regrouping.

Development

For purposes of clarity, the development of these two processes (addition and subtraction) with fractions has been separated. However, the processes should be developed concurrently in the classroom. Subtraction of fractions will be seen as the inverse of addition.

The reading of equations from rod patterns as presented in Curriculum Guide, Pure Number, Section F, page 75, should be continued.

Addition

1. *Like Denominators*

(*a*) $\frac{1}{3} + \frac{1}{3} = \square$

(*b*) $\frac{2}{8} + \frac{1}{8} + \frac{1}{8} = \square$

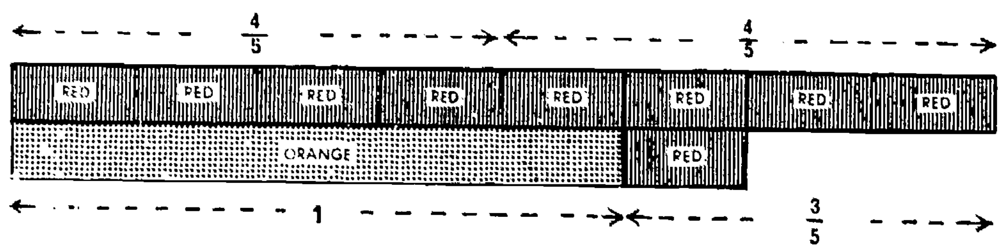
This step should not prove difficult for any child who has reached Section G. It is unlikely that any further concrete experiences will be necessary.

(c) $\frac{4}{5} + \frac{4}{5} = \square$

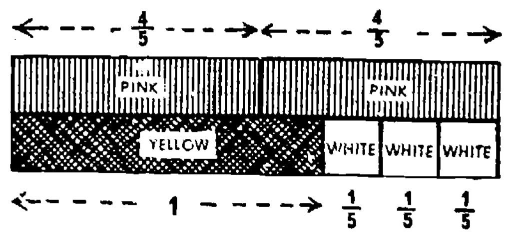
Here the child's answer may involve a fraction in mixed form. This has been discussed above. The actual operation of addition and the resulting total can be shown with rods, number lines, or geometric shapes.

For example :

When "orange" is 1.

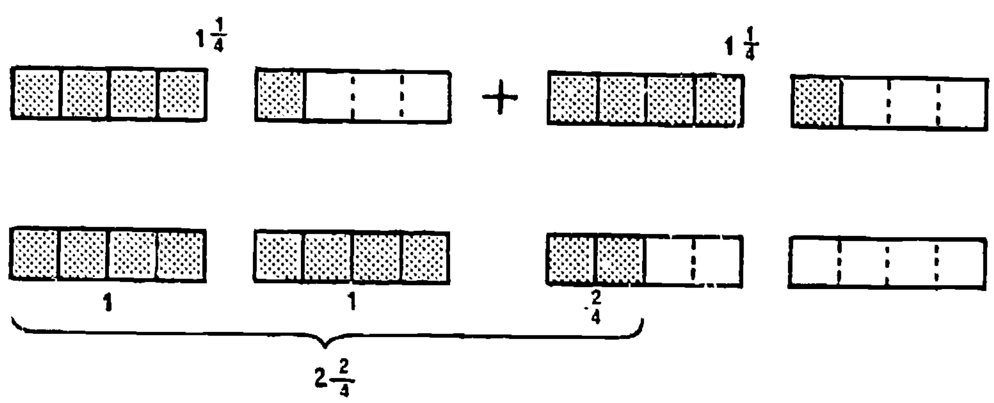


When "yellow" is 1.



(d) $1\frac{1}{4} + 1\frac{1}{4} = \square$

In this step the addends involve fractions in mixed form. This can also be illustrated by a variety of situations. The following is one example.



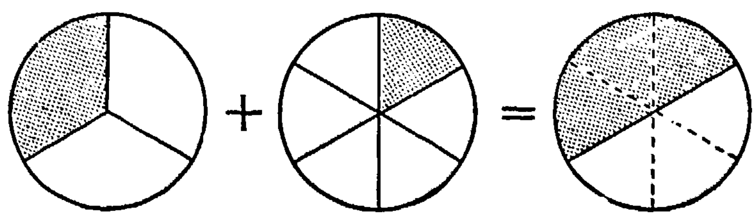
2. *Unlike Denominators (Common Denominator Included)*

Here the child is concerned with fractions that have different denominators. However, these denominators are related to each other in the sense that one is a multiple of the others. The lowest common denominator is included.

Before the child can solve an equation such as $\frac{1}{2} + \frac{1}{4} = \square$, he must realize that fractions with unlike denominators must be renamed in terms of the same denominator. For example, $\frac{1}{2} + \frac{1}{4} = \square$ may be renamed as $\frac{2}{4} + \frac{1}{4} = \square$.

(a) $\frac{1}{3} + \frac{1}{6} = \square$

This example can be shown in terms of geometric shapes. Here, the circle represents the unit of measure.



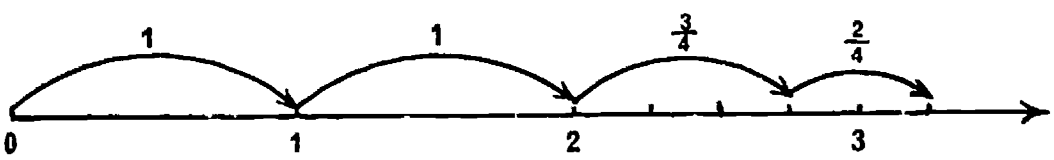
(b) $\frac{9}{10} + \frac{1}{5} = \square$

In this step the answer will be a fraction greater than one.

(c) $1\frac{3}{4} + 1\frac{1}{2} = \square$

This example involves addends that are fractions in mixed form. The solution to such a problem can be successfully shown with a number line.

$$1\tfrac{3}{4} + 1\tfrac{1}{2} = 1 + \tfrac{3}{4} + 1 + \tfrac{1}{2} = 1 + 1 + \tfrac{3}{4} + \tfrac{2}{4}$$



3. *Unlike Denominators (Common Denominator Not Included)*

In examples of this kind the lowest common denominator should not be larger than sixteen. (See Course of Study, Section G, page 10.)

When working with fractions that are not related, the child discovers the common denominator by constructing a table of equivalent fractions.

For example :

$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$
$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$

From this table he establishes that $\frac{1}{3}$ is equal to $\frac{4}{12}$ and $\frac{1}{2}$ is equal to $\frac{6}{12}$. The situation now becomes $\frac{4}{12} + \frac{6}{12} = \frac{10}{12}$.

Here the child discovers that, unlike the situation with "related" fractions, he is required to rename both numbers. This technique arises naturally from the child's earlier work with equivalents. Graded examples might follow the sequence :

(a) $\frac{1}{3} + \frac{1}{2} = \square$

(b) $\frac{2}{3} + \frac{1}{5} = \square$

(c) $\frac{3}{4} + \frac{5}{6} = \square$ (The answer may be a numeral in mixed form.)

(d) $1\frac{2}{7} + 2\frac{1}{2} = \square$ (Addends involving fractions in mixed form.)

Subtraction

Activities with number lines, geometric shapes, and rods as suggested above in regard to addition will also apply to *each* step in subtraction.

1. *Like Denominators*

(a) $\frac{4}{7} - \frac{3}{7} = \square$ (c) $1\frac{1}{3} - \frac{2}{3} = \square$

(b) $1\frac{2}{3} - \frac{1}{3} = \square$

2. *Unlike Denominators (Common Denominator Included)*

(a) $\frac{1}{2} - \frac{1}{4} = \square$ (c) $1\frac{2}{3} - \frac{1}{6} = \square$

(b) $\frac{2}{3} - \frac{1}{6} = \square$ (d) $1\frac{1}{6} - \frac{2}{3} = \square$

3. *Unlike Denominators (Common Denominator Not Included)*

(a) $\frac{1}{2} - \frac{1}{3} = \square$ (d) $1\frac{1}{5} - \frac{1}{2} = \square$

(b) $\frac{2}{3} - \frac{1}{5} = \square$ (e) $2\frac{1}{2} - 1\frac{2}{7} = \square$

(c) $1\frac{5}{6} - \frac{3}{4} = \square$ (f) $2\frac{1}{4} - 1\frac{5}{6} = \square$

In solving numerical examples the child may utilize any of a variety of approaches. No one method is preferred. (Five possible variations in approach are given in the Course of Study, page 11.) In fact, a method that is most suitable in one situation may be least appropriate in another. Teachers should encourage the child to find alternative methods. Skill in this should not be gauged from methods of recording but from oral explanations given by the child.

Numerical Activities

1. Complete—

3 .... + 4 .... = 7 eighths.

8 .... − 4 .... = 4 ninths.

11 twelfths = 6 .... + 5 ....

1 fifth + $\square$ fifths = 4 fifths.

$\square$ third + 1 third = 2 thirds.

2. Complete—

$\frac{1}{3} + \frac{2}{3} = \triangle$ $\frac{3}{7} + \triangle = \frac{8}{7}$

$\frac{7}{9} - \frac{3}{9} = \triangle$ $\triangle - \frac{1}{3} = \frac{2}{9}$

3. Copy and complete each sentence, writing + or − in place of the $\odot$

$\frac{1}{7} \odot \frac{5}{7} = \frac{6}{7}$ $\frac{5}{8} \odot \frac{3}{8} = \frac{2}{8}$

$\frac{3}{5} = \frac{4}{5} \odot \frac{2}{10}$ $2\frac{1}{4} = 1\frac{1}{8} \odot 1\frac{1}{8}$

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4. Copy and complete each sentence using $>$, $<$, or $=$ in place of the $\square$.

$\frac{3}{5} + \frac{4}{5} \square 1\frac{4}{5}$

$\frac{6}{7} - \frac{3}{14} \square \frac{3}{14}$

5. Make use of the following sets of fractions to write as many true statements as you can.

(a) $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}$ e.g., $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

$\frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

$\frac{3}{4} > \frac{1}{2} > \frac{1}{4}$

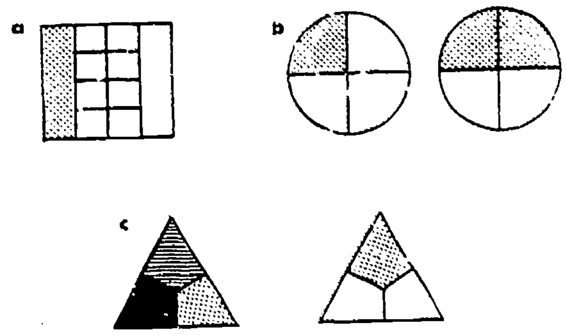
(b) $\frac{9}{10}, \frac{5}{10}, \frac{2}{5}$

(c) $1\frac{3}{5}, \frac{4}{5}, \frac{4}{5}$

6. Match each equation below to a suitable diagram.

$\frac{1}{4} + \frac{1}{2} = \square$

$\frac{3}{4} + \frac{1}{2} = \square$

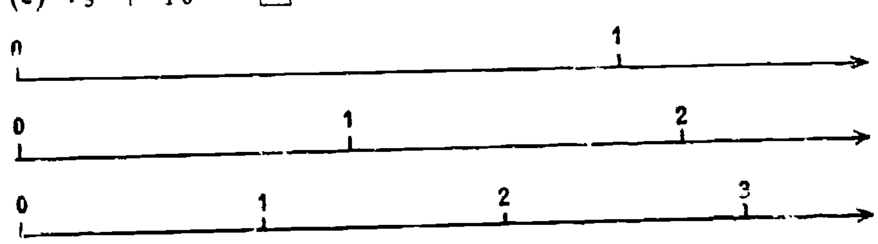


7. On a number line illustrate each of the following incomplete equations :

(a) $\frac{1}{3} + \frac{1}{6} = \square$

(b) $\frac{5}{7} + \frac{9}{14} = \square$

(c) $1\frac{4}{5} + \frac{9}{10} = \square$



8. Show each of the following situations with rods :

(a) $\frac{4}{5} + \frac{3}{10} = \square$

(b) $\frac{1}{2} - \frac{1}{8} = \square$

9. Solve these problems :

(a) Dad has cut $\frac{3}{5}$ of the lawn. How much is left to mow ?

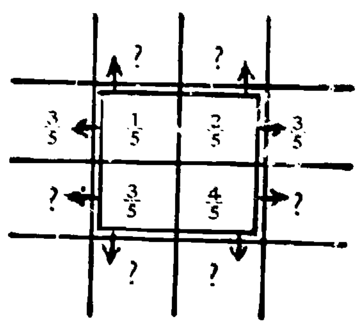
(b) The first fish John caught weighed $1\frac{3}{4}$ pounds and the second weighed $2\frac{1}{8}$ pounds. How much did the fish weigh together ?

10. Complete each table :

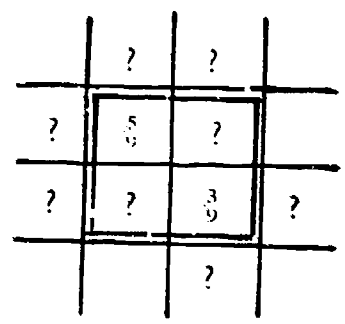
+	$\frac{1}{5}$	$\frac{2}{5}$
$\frac{3}{5}$	?	?
$\frac{2}{5}$	?	$\frac{4}{5}$

—	[illegible]	$\frac{1}{5}$
$\frac{3}{5}$	$\frac{1}{5}$	?
$\frac{4}{5}$	?	$\frac{3}{5}$

Addition



Addition



Subtraction

$\frac{8}{7}$	?	[illegible]
?	$\frac{3}{7}$	?
$\frac{2}{7}$	?	?

11. Complete each of the following tables :

Addends	*Totals*	*Addition Equations*	*Subtraction Equations*	*n*
$\frac{7}{8}$, n	$1\frac{3}{8}$	$\frac{7}{8} + n = 1\frac{3}{8}$ $n + \frac{7}{8} = 1\frac{3}{8}$	$1\frac{3}{8} - \frac{7}{8} = n$ $1\frac{3}{8} - n = \frac{7}{8}$	$\frac{4}{8}$
$\frac{1}{3}$, $\frac{1}{5}$		$\frac{1}{3} + \frac{1}{5} = n$		
			$\frac{5}{6} - \frac{3}{6} = n$	

Equations	*Equivalents*	*Equation*	*n*
$\frac{1}{5} + \frac{1}{2} = n$	$\frac{1}{5} = \frac{2}{10} = \frac{3}{15} =$ $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$	$\frac{2}{10} + \frac{5}{10} = n$	$\frac{7}{10}$
$\frac{4}{7} - \frac{3}{14} = n$	$\frac{4}{7} =$ $\frac{3}{14} =$		
$1\frac{5}{6} - \frac{7}{12} = n$			

12. Applied Number :

$1 \text{ foot} + 1 \text{ foot} = \frac{\square}{\triangle} \text{ yard.}$

$\square$ gal. $= 3$ gal. $+ 3$ pt.

4 oz. $+ \frac{1}{2}$ lb. $= \square$ lb.

C. MULTIPLICATION OF VULGAR FRACTIONS

Aim

To extend the child's understanding of, and skill with, fractions in terms of :

The nature of the operation of multiplication and its relation to other operations ; and

the development of skill in multiplying fractions by whole numbers and whole numbers by fractions—

fractions up to and including twelfths ;

whole numbers to ten.

Notes

The child has different ways of recording and verbalizing multiplication, for instance, "of", "times", "multiplied by", and "×".

In Section G it is expected that the child who uses "of" and/or "times" to record multiplication will progressively replace these symbols by "×". In certain instances, he should read "×" as "multiplied by", for the following reasons :

1. $\frac{4}{3} \times 12 = 16$ cannot be regarded as a case of partition division as might be implied if "×" was read as "of". The partition of twelve articles can never lead to an outcome of 16.

2. $\frac{4}{3} \times 12 = 16$ would be meaningless if we interpreted $\frac{4}{3}$ as "four-thirds times – – –". The word "times" implies a carrying out of the same procedure with repetition. Consequently, in mathematical statements, such phrases as "a half times" and "three and a half times" are meaningless.

Development

1. The child is familiar with a fraction operating upon a whole number. This idea was introduced in Section C (see Curriculum Guide, page 39).

$$\frac{1}{4} \text{ of } 8 =$$

He must now be led to see that this incomplete equation can be written

$$\frac{1}{4} \times 8 =$$

2. The child may be familiar with situations in which a whole number is operating upon a fraction, for instance $8 \times \frac{1}{4} = \square$

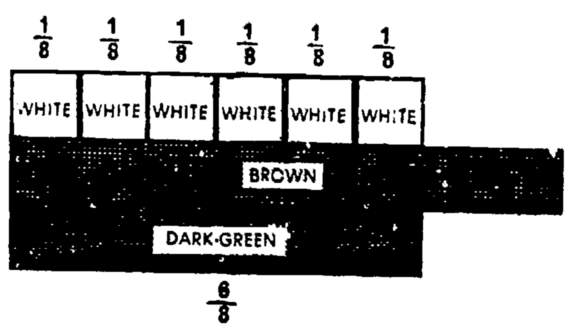
In this example, multiplication may be considered as repeated addition. Thus,

$$8 \times \frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{8}{4}$$

This idea can be developed through the use of rods, number lines, and geometric shapes.

(*a*) *Rods* : $6 \times \frac{1}{8} = \square$

When brown is called one :

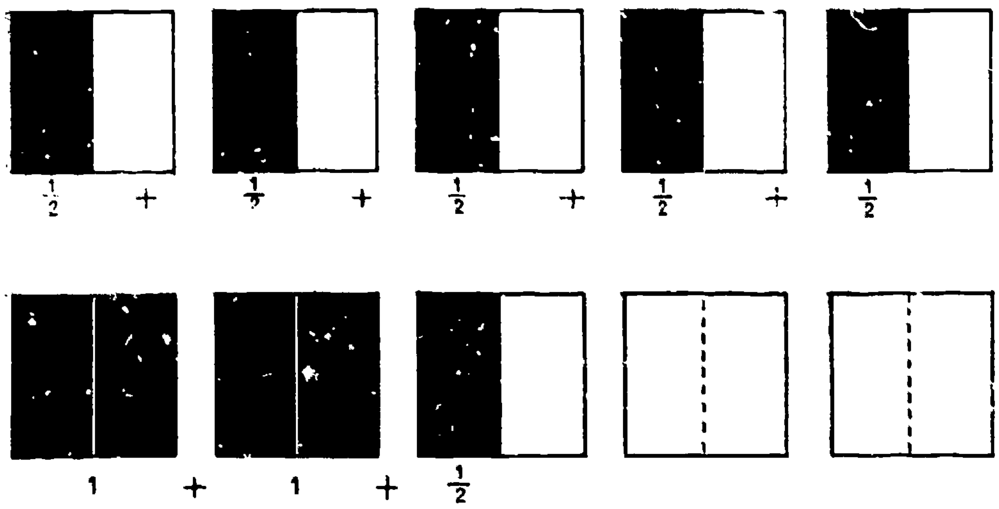


(*b*) *Number Lines* : $8 \times \frac{1}{7} =$

8 jumps of $\frac{1}{7}$

$$\frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} = \frac{8}{7}$$

(c) *Geometric Shapes* : $5 \times \frac{1}{2} = \square$



$\frac{5}{2} = 2\frac{1}{2}$

3. An alternative approach to this aspect of the topic is through an examination of a pattern in number. To solve the example $4 \times \frac{1}{16} = \square$, the child may utilize the technique of halving. He may be encouraged to commence with $4 \times 2 = 8$ and continue until his recording shows the pattern :

$$4 \times 2 = 8$$
$$4 \times 1 = 4$$
$$4 \times \frac{1}{2} = 2$$
$$4 \times \frac{1}{4} = 1$$
$$4 \times \frac{1}{8} = \frac{1}{2}$$
$$4 \times \frac{1}{16} = \frac{1}{4}$$

Initially, care needs to be taken in selecting examples, since this approach is not suitable in all cases. The child will have to gauge the appropriateness of the approach through trial and error.

Note : In this section the computational aspect of the work will be limited. The following examples would be representative :

(*a*) $3 \times \frac{1}{4} = \square$

(*b*) $3 \times \frac{2}{5} = \square$

Activities

Numerical activities in this section will be similar in variety to those undertaken with addition and subtraction, although in this case adapted to multiplication. Variations could be :

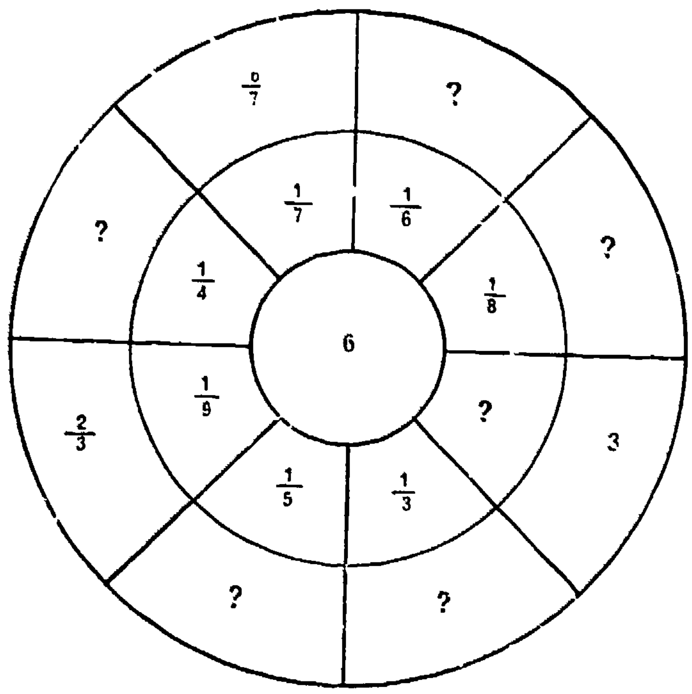
1. $\frac{2}{3} = \square \times \frac{1}{3}$

   $7 \times \frac{1}{2} = \triangle$

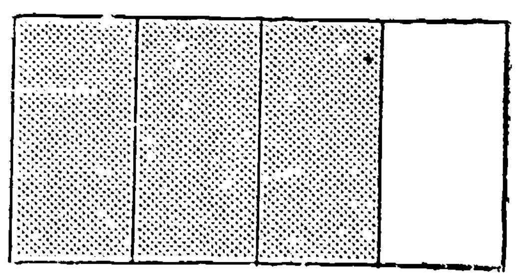
   $\frac{1}{4} + \frac{1}{4} = 2 \times \triangle$

   $2 \times \frac{1}{3} = \square + \square$

2. Multiplication Wheel



3. Name the part of the rectangle that is shaded. Can you give it three different names ?



The frames are given to help you :

□

or △ + △ + △

or ▽ x ⊙

Acceptable answers here would be :
$\frac{3}{4}$, $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$, and $3 \times \frac{1}{4}$

4. Give two other numerals for each of the following :
$\frac{1}{6} + \frac{1}{6}$; $5 \times \frac{1}{6}$; $\frac{5}{7}$.

5. A boy's step measures $2\frac{1}{3}$ feet. How far will he walk if he takes five identical steps ?

D. ADDITION AND SUBTRACTION OF DECIMAL FRACTIONS

Aim

To develop further the child's understanding and skill in adding and subtracting decimal fractions, with examples limited to three addends of units and tenths.

Development

If the approach as given below is adopted it might be best to take this topic late in Section G. This is because it requires a number of skills and understandings that are being built up through other aspects of the child's work. It is an extension of ideas studied with whole numbers, place value, and vulgar fractions.

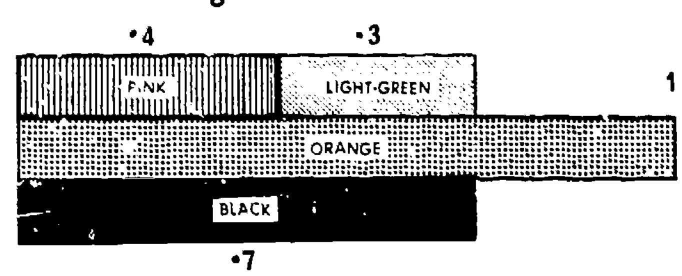
In Section F the child was introduced to the decimal notation form of recording vulgar fractions with denominators of ten. Also, in Section G extensive oral work in counting from one-hundredth to one by hundredths has been undertaken. (See pages 17–18 of this Guide.) On this basis children may examine this topic in the following ways.

1. *Structured Aids*

Rods

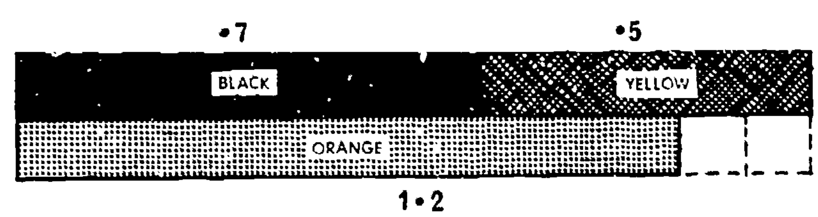
$\cdot 4 + \cdot 3 = \square$

When "orange" is one.



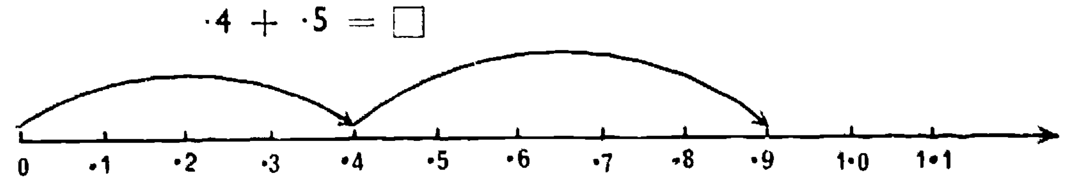
$\cdot 7 + \cdot 5 = \square$

When "orange" is one.



Number line

$\cdot 4 + \cdot 5 = \square$



2. *Renaming Vulgar Fractions in Decimal Notation*

Before children come to work these examples they will have handled addition of vulgar fractions. Exercises can be given which involve $\frac{1}{2}$s, $\frac{1}{5}$s, and $\frac{1}{10}$s. Work done in Section F on the equivalence relationships between these fractions will then be used.

(*a*) 5 tenths + 2 tenths = $\square$ tenths

$\frac{5}{10} + \frac{2}{10} = \square$

$\cdot 5 + \cdot 2 = \square$

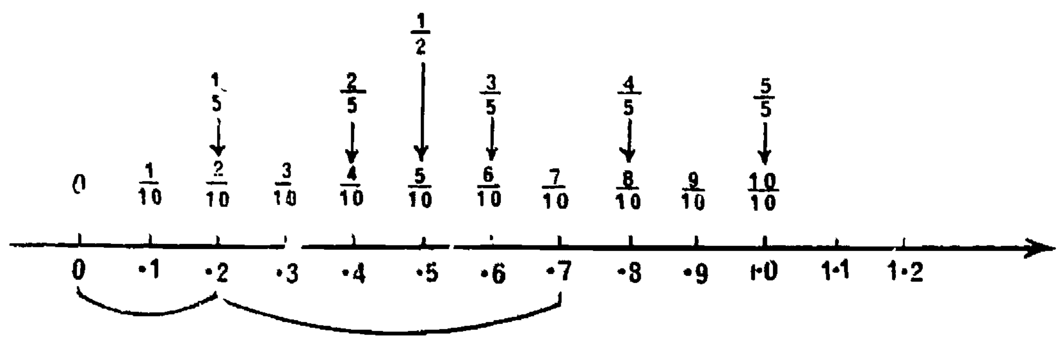
(*b*) $1\frac{4}{10} + \frac{9}{10} = \square$

$1{\cdot}4 + 0{\cdot}9 = \square$

(*c*) $\frac{1}{5} + \frac{1}{2} = \square$

$\cdot 2 + \cdot 5 = \square$

This example might also be illustrated on a number line.



Work similar to that above can be undertaken with subtraction.

3. *Graded Examples*

Addition		Subtraction
$\cdot 4 + \cdot 3 = \square$	(No regrouping)	$\cdot 7 - \cdot 2 = \square$
$1\cdot 7 + 3\cdot 2 = \square$		$3\cdot 8 - 1\cdot 4 = \square$
$\cdot 5 + \cdot 9 = \square$	(Regrouping)	$3\cdot 2 - 1\cdot 2 = \square$
$2\cdot 4 + 3\cdot 7 = \square$		$3\cdot 2 - 1\cdot 9 = \square$
$\cdot 5 + \cdot 5 = \square$	(Zero)	$2\cdot 7 - 1\cdot 8 = \square$
$5\cdot 7 + 3\cdot 3 = \square$		$2 - \cdot 7 = \square$
$3 + 0\cdot 7 = \square$		$2\cdot 4 - 1 = \square$
$1\cdot 4 + 0\cdot 7 = \square$		$5 - 2\cdot 4 = \square$
$1\cdot 5 + \frac{3}{10} = \square$		2 [illegible]/10 $- 0\cdot 5 = \square$
$2\cdot 3 + \frac{1}{5} = \square$		

A sequence similar to that above may be considered when presenting examples of three addends.

Activities

(*a*) Ring the correct answer.

$\cdot 8 + \cdot 6 = \square$ 14, ·14, 1·4

(*b*) Write as many equations as you can, using this set of numerals.

0·7, 0·5, 0·2

To such instruction the child might write—

$0\cdot 5 + 0\cdot 2 = 0\cdot 7$, $0\cdot 7 - 0\cdot 5 = 0\cdot 2$.

(c) Complete the table.

Total	*Addend*	*Addend*
·9	·4	
	·7	·3
·5		·3

(d) Solve these problems.

How much greater is 1·5 minutes than ·4 minutes ?

Mr. Andrew bought 5·4 gallons of petrol on Monday and 4·3 gallons on Friday. How much petrol did he buy ?

(e) Copy and complete each sentence using $>$, $<$, or $=$ to make it true.

$\cdot 9 \; \triangle \; \cdot 3 + \cdot 7$

$1 \cdot 5 - \cdot 6 \; \triangle \; 1 \cdot 0$

$3 \cdot 4 \; \triangle \; 4 \cdot 4 - 0 \cdot 9$

E. MULTIPLICATION OF DECIMAL FRACTIONS

Aim

To develop understanding and skill in multiplying decimals by whole numbers and whole numbers by decimals. Generally the multiplier should be one of the following :

1, 2, 3, and so on to 12.

100, 1000.

·1, ·2, ·3, and so on to ·9.

Development

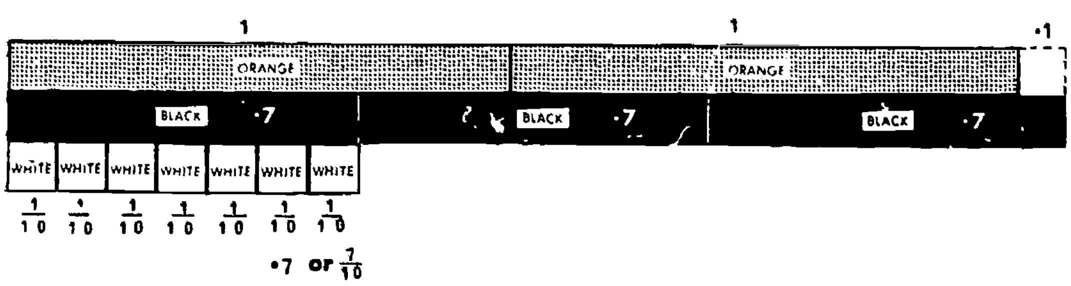
1. The initial approach may be made through the relationship between the operations of multiplication and addition.

$3 \times \cdot 7 = \cdot 7 + \cdot 7 + \cdot 7$

Note that in this example the assumption is that the child is reading the sign " × " as "times". See Curriculum Guide, Section F (page 89) for a discussion of this point.

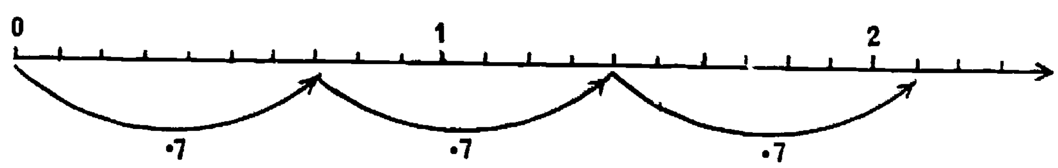
Use can be made of structured aids and semi-abstract or abstract situations to extend the child's understanding. The first example given makes use of the rods by naming the orange rod as one.

When "orange" is called 1.



Base 10 material such as Dienes's Multibase Arithmetic Blocks can be very useful in working examples such as $3 \times \cdot 7 = \square$.

The same example can be illustrated on the number line.



With numerals, the situation can be examined in the following ways :

$3 \times \cdot 7 = \square + \square + \square$ $\qquad$ $3 \times \cdot 7 = \square$ tenths $+ \square$ tenths $+ \square$ tenths

$= \square$ tenths

$= \nabla$

$\cdot 7 + \cdot 7 + \cdot 7 = \square \times \triangle$

$= \odot$

70

2. The child's knowledge of multiplication of vulgar fractions by whole numbers can also be utilized.

$$\begin{aligned} 3 \times \cdot 7 &= 3 \times \tfrac{7}{10} \\ &= \tfrac{21}{10} \\ &= 2 + \tfrac{1}{10} \\ &= 2 \cdot 1 \end{aligned}$$

This approach is much more meaningful than the one where the child learns a mechanical rule such as : "Count the number of digits after the decimal point in the multiplier and the multiplicand and put this number of digits after the decimal point in the product." Such a mechanical device for placing the decimal point may eventually be adopted, but at this stage in the child's mathematical development it is inappropriate.

3. An important skill that should be developed at this stage is estimation of products. The child's success with this skill will depend mainly upon his ability to "round off". In an example such as $3 \cdot 7 \times 5$, $3 \cdot 7$ can be seen as approximately 4, so the answer should be about 4×5, which is 20. Any product that is close to and less than twenty could be accepted as being plausible.

The child should be given practice in approximating above and below to the nearest whole number. See page 50 of this Guide.

For example : $3 \cdot 9$ is about 4 .
$6 \cdot 3$ is about 6 .

It is normally agreed to *round upwards* any decimal fraction terminating in 5.

Example : $7 \cdot 5$ is about 8 .

4. Another useful skill that can be developed at this stage is finding a product when one of the factors is 10 or a power of 10. The child will discover through his examination of the emerging pattern that the answer can be obtained by relocating the decimal point according to the power of 10 that is used as the other factor :

For example :

$4200 \times 10 = 42000$	$4200 \times 100 = 420000$
$420 \times 10 = 4200$	$420 \times 100 = \boxed{}$
$42 \times 10 = \boxed{}$	$42 \times 100 = \boxed{}$
$4 \cdot 2 \times 10 = \boxed{}$	$4 \cdot 2 \times 100 = \boxed{}$

In such examples the teacher is appealing to the child's appreciation of pattern in number.

Activities

The form of the activities listed for addition and subtraction of decimals is also suitable for multiplication of decimals.

MEASUREMENT

AIMS

1. To widen the child's experience and understanding in estimation and measurement in the topics of length, weight, capacity, time, volume, and area.

2. To develop skill in the practical aspects of measurement.

COMMENTS

1. As the same general ideas underlie the various measurement topics, the major ideas after discovery in any one topic are reinforced at a later date when another topic is examined. It is important that the child becomes aware of, and appreciates, this linkage.

2. Some key ideas that should be grasped by the child as a result of experiences in measurement are the following :

(a) *The Nature of Measurement*

Because of its nature, measurement involves a comparison with a given unit. This comparison is a direct one with length, area, volume, capacity, and weight.

(b) *The Unit of Measure*

(i) To compare two quantities, the same unit must be used to measure both. For example, it is not appropriate to compare the area measures of two tiles if a square with a side of 1 inch is the unit of measure in one case while a triangle with each side 2 inches long is the unit of measure in the other.

(ii) The unit of measure adopted must be suitable for the kind of measurement being made. For example, a book may provide a suitable unit for a measure of weight but will not be suitable for measuring time. A triangular shape may be suitable as a unit of area measure but not as a unit for measure of volume.

(iii) If a standard unit is agreed upon in measurement, then communication of information about measures is simplified. A standard unit is one that is generally accepted. The legal definitions of most units of measures will be found in *Statutory Rules*, 1961, No. 142, available from the Commonwealth Government Printer. Nevertheless, it is not intended that children shall memorize definitions from these *Statutory Rules*.

(iv) The choice of a unit of measure is an arbitrary one. This is illustrated by the fact that quantities of the same kind can be measured in different standard units—for example, miles or kilometres, gallons or litres, acres or hectares.

(v) Assuming correct use and interpretation of the measuring instrument, the choice of a smaller unit of measure implies that a greater precision of measurement is being attained.

Most of these understandings have been developing gradually since the child commenced school. However, it is important that these matte should continue to be discussed and reinforced through activities.

3. While there are many common ideas running through the various topics grouped under the heading "Measurement", teachers will need to bear in mind that there are different degrees of difficulty involved in the understanding and the description of the subject-matter of each topic. This awareness may influence the order in which the topics are treated, but the underlying approach will be the same.

LENGTH

AIMS

1. To increase the child's familiarity with length units of measure (inches, feet, and yards) so that—

   (a) estimation of the measure of length in environmental situations shows an increasing accuracy ;

   (b) ideas of precision in measurement for units (yard, foot, inch, and fractions of the inch to $\frac{1}{10}$ inch) are understood and applied.

2. To introduce the "mile" and the "chain" as standard units of measurement and to progressively increase the child's familiarity with them as in (1) above.

3. To develop understandings of relationships existing within the tables and as set out in the Course of Study.

NOTES

1. (a) The child should be encouraged to employ elements in his environment to assist him in arriving at estimates of length. Thus the width of a brick or the height of a door can be used to assist in the estimation of the height of a wall. Estimation should generally precede measurement. Length measures considered should include those of curves, perimeters, and circumferences.

(b) The child should be able to measure and record measurements to the nearest tenth of an inch, when required to do so. He should be able to record measurements with precision to $\frac{1}{8}$ of an inch, $\frac{1}{4}$ of an inch, $\frac{1}{2}$ an inch, 1 inch, 1 foot, 1 yard, 1 chain, and 1 mile, in appropriate cases. Thus an actual measurement of $10\frac{9}{16}$ inches should be recorded as—

$10\frac{6}{10}$ in. (correct to $\frac{1}{10}$ in.), $10\frac{4}{8}$ in. (correct to $\frac{1}{8}$ in.),
$10\frac{2}{4}$ in. (correct to $\frac{1}{4}$ in.), $10\frac{1}{2}$ in. (correct to $\frac{1}{2}$ in.),
11 in. (correct to 1 in.), 1 ft. (correct to nearest ft.).

2. The child's understanding of, and familiarity with, the mile and the chain should be developed as in (1) for yards, feet, and inches.

3. Relationships should be discovered as a result of situations out of which problems arise. Such situations will generally be contrived for the specific purpose of pointing up relationships. Thus a child may be asked to estimate the number of pieces of tape, each 6 inches long, that can be cut from a piece one yard long. By the end of this section the child should be familiar with relationships such as the following :

12 inches = 1 foot	$1\frac{1}{2}$ in. = $\frac{1}{8}$ ft.	6 in. = $\frac{1}{6}$ yd.
3 feet = 1 yard	3 in. = $\frac{1}{4}$ ft.	9 in. = $\frac{1}{4}$ yd.
22 yards = 1 chain	4 in. = $\frac{1}{3}$ ft.	18 in. = $\frac{1}{2}$ yd.
80 chains = 1 mile	6 in. = $\frac{1}{2}$ ft.	1 ft. 6 in. = $\frac{1}{2}$ yd.
		11 yd. = $\frac{1}{2}$ chain.

Although these relationships should be discovered primarily as a result of direct activities, opportunities should be taken to reinforce them through work in pure number as, for example, in fractions. It should be noted, too, that pure number ideas can well be reinforced, and even developed, through environmental situations that may appeal more to some children than do other means of presentation.

4. Problems should involve, at various times, practical investigation, theoretical consideration, simple computation, and opportunities for a variety of methods of solution. Complex computation, divorced from the child's reality, should be avoided. Generally, more than two different units should not be employed to measure a quantity ; thus a length would not be recorded in chains, yards, and feet, but rather in chains and yards, or in yards and feet.

DEVELOPMENT

Children's activities should be firmly centred on practical situations. Formulae (for reduction, perimeter, or circumference) should not be presented. This is not to say that where a child develops these for himself he should be discouraged.

Encouragement should be given to the child to discover for himself his own techniques of measurement. Teachers can assist such discoveries by making suitable materials freely available and by encouraging group discussion of problems. Materials supplied could include lengths of string, map-wheels, trundle-wheels, calipers, measuring tapes, and rulers.

Individual standards for comparison to facilitate estimation should be developed. To this end it is useful to consider the heights or the lengths of elements common to many environments, such as the following : the length of a brick ; the lengths of one's pace and one's hand span, and the distance from finger tip to finger tip with arms outstretched ; the lengths of a cricket pitch, a football ground, and a basket-ball court ; the time taken to walk one mile ; and the number of steps taken to walk a mile. All of these are standards that the child can gradually build up to assist him in the estimation of length. Thus a reasonable approximation to the height of a wall can be obtained if the number of rows of bricks used in its construction can be counted. Links between the body parts and bodily movement, together with the development of standard units of length measure (for example, the fathom, the Roman double pace, the cubit), can be introduced here as enrichment material.

Introduction of the Chain and the Mile

To introduce the chain the teacher could well mark out a chain in the school-ground. A rope, one chain long, can be used to measure lengths such as the perimeter of the school-ground.

The child should estimate lengths in chains and round off measures in chains. The historical usage of the chain can be introduced. After the child has become familiar with the chain and has attained some accuracy in estimation of distances of one chain (say with error to the order of two or three yards) he can measure the marked chain in yards. Further measurement activities should be provided for the child to measure in chains and yards.

Techniques to facilitate estimation in chains may include the use of the wheel (e.g. bicycle), paces, spaces between telephone posts, and so on. Children should be encouraged to discover the distance in chains from home to school.

The mile should be introduced as the distance between two points in the child's environment—say two mile posts or other permanent landmarks such as street corners, trees, or buildings. Some form of odometer such as a car "speedometer", a trundle wheel, or a cycle mile meter can be used to measure other environmental distances of one mile. Children should use personal techniques to assist them to estimate a mile—techniques such as the number of paces taken or the time taken to walk one mile. It does not seem useful to measure a mile in chains directly. Later, children may remeasure a measured quarter-mile in chains, using a chain tape or rope, or they may measure such lengths in yards using a trundle wheel. The child's knowledge of fractions and multiplication can be utilized to establish relationships between yards and chains and between chains and miles. Hence a quarter-mile measured as 20 chains implies that a mile measures 80 chains. Children should have much experience in actual measurement in a variety of situations. Problems should cater for environmental activity, analysis of elements appropriate to the child's ability, and very simple computation.

Some discussion should centre on estimation in terms of best over-estimate or best under-estimate of a length. This discussion could well be introduced through situations such as the packing of material into given containers and finding containers to fit given materials, fitting glass into frames, buying clothes, shoes, and the like. (An opportunity exists here for a discussion of the value of "best under-estimate" in long division.)

As outcomes of experience in estimation and measurement, children should understand ideas of "rounding off", understand ideas of best estimates for particular purposes, and recognize stated measurements of familiar lengths as reasonable estimates or otherwise. Thus in rounding off to a larger unit, the following may be expected : $3\frac{3}{4}$ inches as 4 inches ; 2 feet 3 inches as 2 feet ; 478 miles 13 chains as 478 miles. It is important that each situation should be examined to ascertain whether the rounding off should be taken to the next higher or the previous lower measure.

SUGGESTED ACTIVITIES

1. (a) How many paces do you take to walk 22 yards ?
   (b) Estimate the number of paces you would take to walk one mile.
   (c) Walk one mile and check your estimate.

2. (a) Estimate, in yards, the length of the diagonal from corner to corner of a basket-ball court.

(b) Step out this distance and reconsider the estimate you have made.

(c) Measure the distance using a "chain" tape.

3. (a) How many minutes does it take for you—

(i) to walk a half-mile,

(ii) to jog a half-mile ?

Count your steps each time.

(b) Estimate the length of your stride when running.

4. Estimate the perimeters of circular objects in the classroom and check your measures, say, by using a length of string or by rolling the object. (The child should be encouraged to find these and other ways for himself.) Estimate diameter lengths. Measure these and compare your answer with that for the perimeter.

5. Estimate the length of the boundary of an irregular shape. Check the estimation, using a wheel or a piece of string.

6. Choose two points in the school-ground as far apart as possible and in a direct line. How many times will you have to walk this distance to cover one mile ? What fraction of a mile (approximately) is this distance ?

Examples of this type may well be used to enrich children's ideas of addition and multiplication of vulgar fractions.

Example :

If 16 lengths $=$ 1 mile approx. then 1 length $= \frac{1}{16}$ mile approx.
and 8 lengths $= \frac{1}{2}$ mile approx. or 40 ch. approx.
and 4 lengths $= \frac{1}{4}$ mile approx. or 20 ch. approx.
and 2 lengths $= \frac{1}{8}$ mile approx. or 10 ch. approx.
and 1 length $= \frac{1}{16}$ mile approx. or 5 ch. approx.

$$\tfrac{1}{8} + \tfrac{1}{8} + \tfrac{1}{8} + \tfrac{1}{8} = \tfrac{4}{8} = 40 \text{ chs.} = \tfrac{1}{2} \text{ mile.}$$

$$\begin{cases} \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \\ \frac{1}{2} \text{ of } 40 \text{ ch.} = 20 \text{ ch.} \end{cases} \qquad \begin{cases} \frac{1}{2} \times \frac{3}{8} = \frac{3}{16} \\ \frac{1}{2} \text{ of } 30 \text{ ch.} = 15 \text{ ch.} \end{cases}$$

AREA

AIM

To provide experiences that will further the child's understanding of area and its measurement in terms of the following :

(*a*) A surface has two dimensions. Examples of surfaces of different kinds are found in cones, cylinders, and plane (i.e. "flat") regions.

(*b*) Two or more surfaces can be compared in area.

(*c*) Regions having different shapes can have the same area.

(*d*) The unit for measuring a surface is an area unit.

(*e*) Some unit shapes are suitable for the measurement of certain regions, others are not.

NOTES

Owing to the stress placed on the understanding of surface measurement, no work involving formal units of measurement is included in this section. This understanding is assisted by the child's direct experience with concrete materials.

DEVELOPMENT

1. Models of surfaces can be used to develop the notion that an area has the dimensions of length and width but no depth or height. The child can be led to consider labels on tins, dust on surfaces, coats of paint, book jackets, or moisture over a surface in this way.

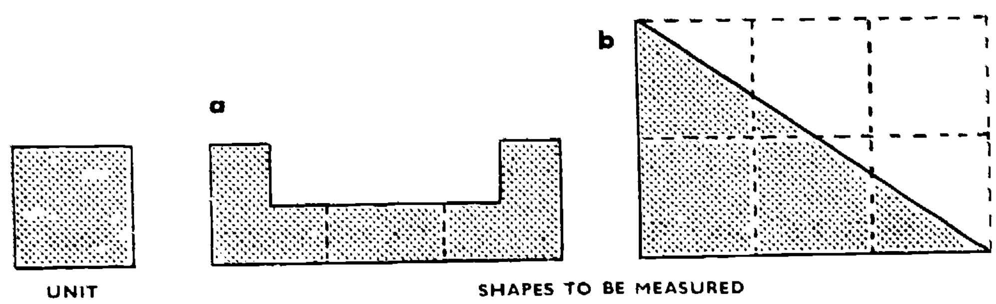
2. Techniques for comparing surfaces should range from cutting and rearrangement to the direct application of informal units of measure of area such as small tile-shaped counters. The value of a standard unit for comparison should be realized when the child attempts to communicate the result of measurement. Estimation of a result should be practised before comparisons are made by other means. Where a pattern is seen on the surface under consideration, this pattern should be used to help to make the comparison. For instance, floor tiles, brick faces, chequered patterns, window-panes, and wall-paper patterns are useful aids. In some cases, a prepared graticule can be used. This aid consists of a transparent sheet on which a regular pattern is inscribed. The sheet is placed over the shapes to be compared and the appropriate units are counted. The use of such a sheet can also be valuable in assisting the child to realize that some shapes are suitable as units for measurement while others are not. Thus it will be discovered that units shaped as equilateral triangles are not suitable for measuring shapes whose boundaries are rectangles and that units based on sectors of circles have very limited use.

The superposition of uniformly shaped objects such as postage stamps, infant room inch squares, building blocks, and newspaper sheets is a useful activity that leads to a simple means for comparing the areas of regions, to a greater understanding of surface measurement, and to the realization that certain shapes are more suitable than others for measuring regions of various shapes.

It should be kept in mind that no standard formal unit is introduced to the child at Section G level.

This is not to say that such units are not to be used informally. On the contrary, it is desirable that the child should have experience in area measuring with inch squares or sheets ruled into inch squares.

It is important that the child should cut and rearrange chosen unit shapes in order to measure shapes that may not be readily measured by using the given square-shaped unit.

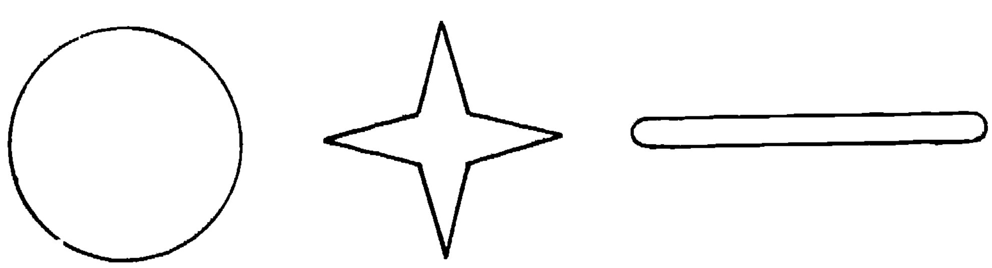


Attention should be focused on the fact that a unit piece is unchanged in its area even though it is cut and rearranged.

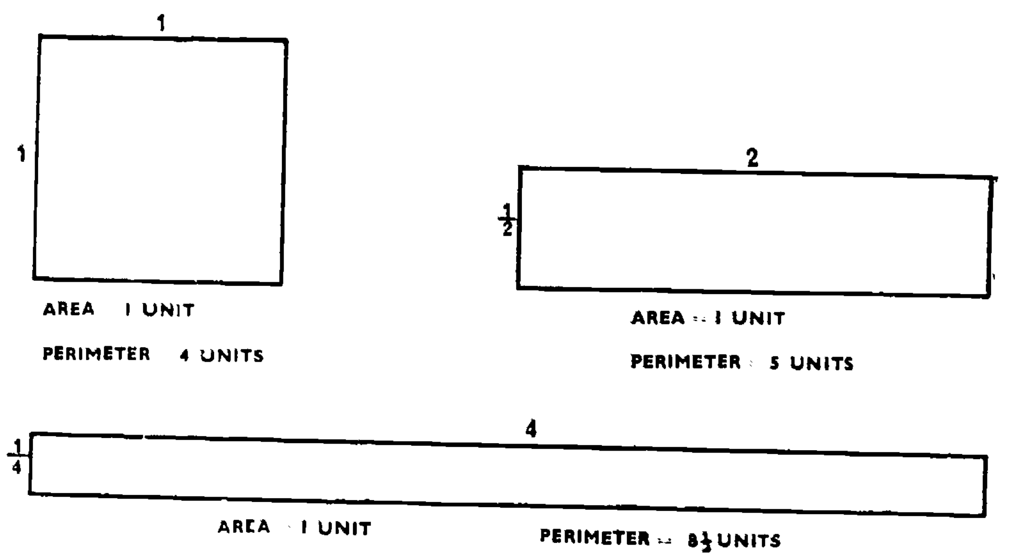
When the child is able to compare and measure surfaces, using informal units, and to communicate the results of his measurements with confidence, attention could well be directed towards the non-relationship between area and perimeter. Through practical exercises the child may become aware that—

(*a*) changing the shape of a surface by cutting and rearranging does not increase or decrease its area but the perimeter *can* vary, and usually does so ;

(*b*) given a constant perimeter, a great number of shapes can be obtained. There is no constancy of area measure in the shapes. This can be readily seen by using a looped cord.

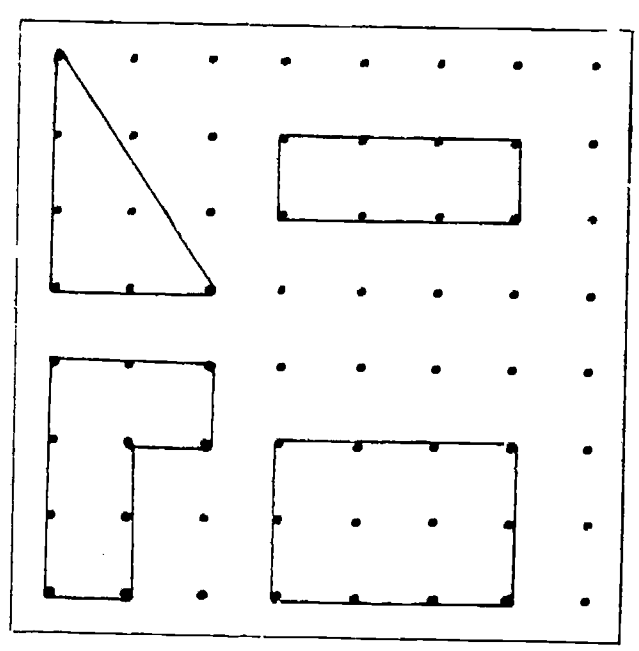


The length of cord is constant, the perimeters are equal, but the area measures are quite different.



Simple activities involving the manipulation of inch squares can be undertaken to reinforce these ideas. For example, a child may be asked to discover shapes having an area of 12 squares with perimeters of 26 inches, 14 inches, 18 inches, and so on.

The geoboard, using rubber bands for constant area situations and looped cord for constant perimeters, can be used in a way similar to that suggested above for the squares.



(I) Equal areas, different perimeters.

(II) Equal perimeters, different areas.

Many possibilities for the integration of spatial relations and area measure exist at this level. In the interests of child understanding and economy of time, these opportunities should be taken up as they arise.

VOLUME

AIM

To develop further the child's appreciation that—

(*a*) volume is concerned with filling space ;

(*b*) shape can be changed without any corresponding change in volume ;

(*c*) volumes can be compared and measured ;

(*d*) weight is not necessarily an indicator of volume.

NOTES

It should not be expected that any of the above ideas, though understood by the child, will be expressed formally by him. It is expected that the child will be provided with challenging problems involving the use of concrete materials and that, out of these situations, he will realize that for particular volumes each of the attributes of volume set out above does hold. The understandings already built up of relationships between volume and capacity of vessels should be maintained. The internal volume (capacity) of vessels should receive close attention. Teachers should satisfy themselves that the child has achieved an understanding of conservation of volume—

(*a*) that a volume of liquid remains unchanged when poured into vessels of different shape ;

(*b*) that materials which are not, under normal conditions, compressible (such as water) neither increase nor decrease in total volume when separated into parts or when a change of shape occurs.

Comparisons of volumes can be made in this section for—

(*a*) the internal volumes (capacities) of vessels or containers such as cups, vases, spoons, boxes, tins ;

(*b*) plastic materials whose shape can be changed to facilitate comparison—clay, plasticine, or dough would be suitable ;

(*c*) fluid materials such as sand or liquids ;

(*d*) simple rectilinear shapes that can be broken down to comparable units—the units could be rectangular prisms, triangular prisms, or pyramids.

Comparison should at first be attempted by estimation, and concrete means should be available for subsequent direct checking.

DEVELOPMENT

Many activities suggested below are revisions and extensions of those suggested in Sections B to F. Activities in volume should be related to corresponding ideas in capacity and spatial relations. Frequently "volume" will arise in discussions associated with particular shapes. Where links occur naturally they should be exploited. For instance, an attribute of rectangular prisms is the efficiency with which they can be packed. This same attribute facilitates the measurement of volume. Furthermore, both capacity and volume are concerned with the ability of a body to occupy space.

The use of rubber moulds to make plaster or plasticine objects can assist the child to understand the space-filling idea of volume. Ice removed from simple containers (bowls, cones, and other kitchenware) can have similar effect.

In relating volume to capacity we may compare the capacities of two vessels by tipping water (or sand, wheat, etc.) from one to the other or by expressing the capacity of each vessel in a selected unit and so making a comparison. Much experience should be obtained by the child in comparison and ordering of volumes. Vessels of many shapes should be made available. Among these should be vessels that are tall-narrow and short-squat, concave-shaped and convex-shaped, thick-walled and thin-walled, pyramid-shaped and box-shaped, and some vessels of specific capacity or volume such as pint, quart, and gallon sizes, and cubic centimetre, cubic decimetre, cubic inch, and cubic foot sizes.

Experiences involving objects of various shapes and volumes could include—

(*a*) filling a large container using one of smaller size ;

(*b*) filling a number of smaller containers from a large vessel ;

(*c*) endeavouring to select containers of equal capacity, and

(*d*) examining commercial containers for indications of size (ounces, fluid ounces, millilitres, and so on) and considering such, making comparisons.

From such experiences, the child will become aware that—

(*a*) a surprising number of small units is needed to fill one of large size ;

(*b*) a relationship exists between the shape of a vessel and its capacity ;

(*c*) capacity cannot always be gauged from apparent size ;

(*d*) containers are labelled in a variety of ways, some of which indicate volume or size while others give little indication of size.

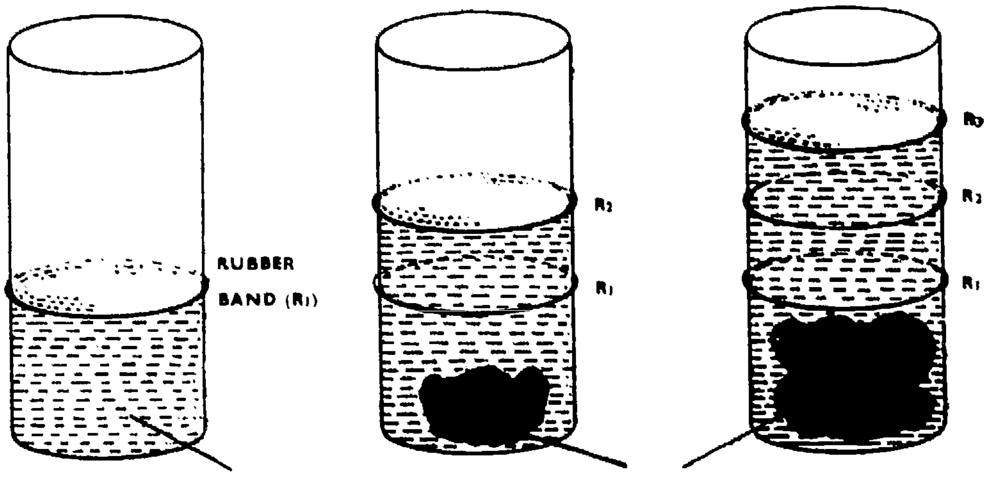
By taking a lump of plastic material such as clay or plasticine and modelling it into different shapes, the child will come to understand that shape can be altered while the volume is conserved.

The "space occupying" property of substances can well be seen when a solid is immersed in a glass or plastic flask of coloured water.

By immersing equal volumes in a glass vessel containing water it is seen that the water level rises by the same amount each time. Blocks of uniform size (provided they are not buoyant) can well be used here.

If two or more blocks are used, the idea of conservation of volume is emphasized since the actual shapes of the submerged blocks have no bearing on the level to which the water rises in the jar. Again this is discovered using plasticine. Plasticine is immersed in water and the water level noted. The plasticine is remoulded, or cut into pieces and immersed again, and the water level is noted to be the same as before.

The problem of comparing the volume or space occupied by irregularly shaped objects should be raised with the class. A likely outcome is the use of the apparatus mentioned above. Estimation of the result should be made before the testing.



Attempts should not be made to measure in formal units. The objective here is only to compare.

In earlier sections (B, C, and D) the child has been concerned with estimation of weight and its subsequent measurement. He has some notions of possible relationships between weight and size. Experiences involving the estimation of weight associated with size and shape of various objects should be continued. With irregular objects, such as stones or screws, volumes can be compared using the "displacement of water" technique suggested above. Weight can be measured using scales. A variety of materials should be investigated, for example, iron, balsa wood, aluminium, hardwood, and plastic.

For some time the child has been having experience in packing objects into containers. This experience has been directed to the packing of rectangular prisms into rectangular boxes. Such experience can well be continued and linked with estimation as to the number of blocks required to fill the box. Informal material (such as matchboxes, shoe-boxes, play blocks) is interesting and essential. A more formal aid, consisting of, say, 1,000 blocks of uniform size, is now needed to develop fully the idea of a solid considered as a collection of blocks.

These blocks are readily available (size, 1 cubic centimetre, though other sizes would be just as suitable). For this development of volume it is essential that the child has frequent access to the material and directed experience in its use.

Example :

A child is given a pile of blocks (say, 27) and he is directed to make a rectangular block using as many of the blocks as possible. He is then asked to write in the form of an equation what he has done. He may write :

$(5 \times 2) + (5 \times 2) + 7 = 27$ . . . (block 5 x 2 x 2 ; 7 unused)
or $4 \times 3 + 4 \times 3 + 3 = 27$ . . . (block 4 x 3 x 2 ; 3 unused)
or $8 + 8 + 8 + 3 = 27$ . . . . . (block 4 x 2 x 3 ; 3 unused)
or $8 \times 3 = 27 - 3$

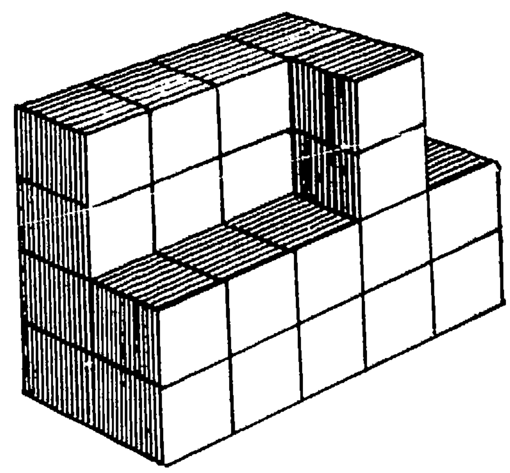
The next direction could well be to construct a cube, using as many of the blocks as possible. The equations recorded might be :

$$3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 27$$
$$\text{or } 3 \times 3 + 3 \times 3 + 3 \times 3 = 27$$
$$\text{or } 9 + 9 + 9 = 27$$
$$\text{or } 3 \times 3 \times 3 = 27$$

An activity such as the following can now be taken : "Build a shape of your own and write equations to describe how you count your blocks."

A child's result might be :

$$6 + 6 + 6 + 8 + 4 = 30$$
$$\text{or } 10 + 10 + 5 + 5 = 30$$
$$\text{or } 5 \times 4 + 2 \times 5 = 30$$
$$\text{or } 5 \times (2 + 4) = 30$$



Further developments based on this activity are as follows : A child is given a pile of 64 blocks and assignments such as :

(*a*) (i) From these blocks make a cube using all the blocks.
  (ii) How many units of length are in each edge of the cube ?
  (iii) How many units of area are enclosed by the edges of each face ?
  (iv) How many storeys or layers high is the cube ?
  (v) How many small cubes are in each level ?

(*b*) (i) Make a rectangle with four rows of eight blocks.
  (ii) How many blocks have you used ?
  (iii) Make a second layer or storey.
  (iv) How many blocks have been used altogether ?
  (v) In how many ways can you count these blocks ? (Illustrations of the commutative laws of addition and multiplication.)

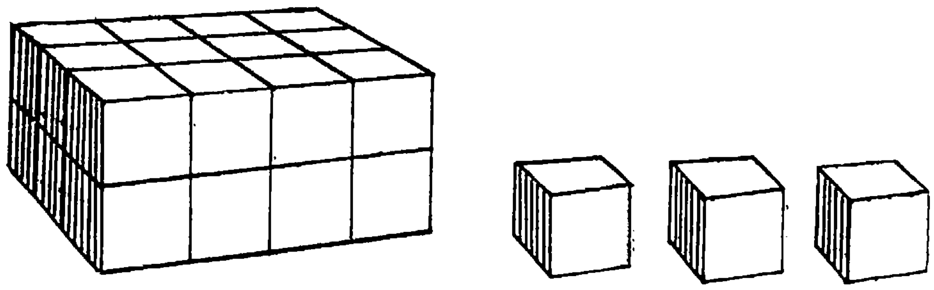
(*c*) (i) Can you make any other rectangular solids using all the blocks ? Make one.
  (ii) Count your blocks and write down the calculation you have used. These might be :

$$16 \times (2 \times 2) =$$
$$\text{or } 16 + 16 + 16 + 16 =$$
$$\text{or } 16 \times 4 =$$
$$\text{or } 4 \times 16 =$$

and so on.

Using 27 blocks, equations that can be obtained include :

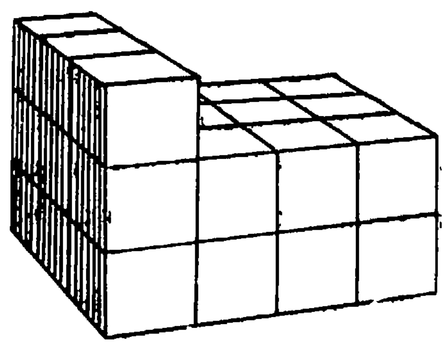
$$\left.\begin{aligned}&(3 \times 3) \times 3 = 27\\&3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 27\\&27 - 3 - 3 - 3 - 3 - 3 - 3 - 3 - 3 - 3 = 0\\&27 - 3 \times (3 + 3 + 3) = 0\end{aligned}\right\}\text{for cube}$$



$3 \times 2 \times 4 = 27 - 3$

or $3 \times 2 \times 4 + 3 = 27$

or $3 \times 3 + 3 \times (3 \times 2) = 27$



In addition to the development of the ideas that—

(a) a solid may be considered as being made up from a collection of blocks of uniform size, and

(b) the shape of a solid may be changed but, if the same blocks are used, the volume is unchanged,

the above is an activity that provides a useful integration with pure number—especially in the creation and the development of equations and in the use of the basic properties. Activities can become as complex as possible for the child to handle—his own created model serves both as a challenge and as a check. Some children will rapidly arrive at the most satisfactory way of counting blocks and, for rectangular prisms, may calculate the number of blocks used rather than count them. The important and necessary result to emerge from experience at this stage is a realization that three-dimensional objects can be regarded as being made up from a number of small units, and that in whatever way these units are arranged their number does not alter—a great variety of shapes can have equal volumes.

CAPACITY

AIMS

1. To provide personal experience in estimation and measurement within the limits of tables set out below under "Units of Measure". Within the same limits to carry out one-step reductions with related fractional work, e.g.

$$1 \text{ pint} = \tfrac{1}{8} \text{ gallon}$$
$$\tfrac{1}{2} \text{ pint} = \tfrac{1}{16} \text{ gallon}$$
$$\tfrac{1}{4} \text{ pint} = \tfrac{1}{32} \text{ gallon}$$

2. To relate the formal aspects of capacity to everyday experience through practical problems.

NOTES

Capacity, like volume, should be seen by the children as relating to the ability of material to fill space ; however, the kind of material measured in gallons or pints is usually a fluid. Other comments on the aims arise incidentally in the development below.

DEVELOPMENT AND PRACTICAL EXPERIENCE

1. Practical Experiences

Experience in comparison, estimation, and measurement of capacities, using suitable containers, should be continued as in Section F. Many different containers of all shapes and sizes should be used. Graduated containers of either metal or plastic should be provided. Even if graduated containers are readily available, all children should be given the opportunity to graduate an unmarked container for measuring pints and gallons. A plastic container could be graduated into pints by using a pint bottle as a measure. For larger quantities, a plastic bucket could be filled with water, using the pint measure to graduate the bucket into gallons.

Such activities as suggested above are suitable for either individual or group experience.

2. Units of Measure

2 pints = 1 quart
4 quarts = 1 gallon
8 pints = 1 gallon

The use of the term "quart" should not be stressed unnecessarily. Only on a very few occasions is the word used in everyday life, for example, a quart of oil. In most situations today only pints and gallons are used. As a result we speak of 2 pints of milk rather than 1 quart of milk. Hence practical exercises in capacity should be concerned primarily with pints and gallons ; however, a child should realize that one quart is equivalent to one-quarter of a gallon, that is, two pints.

The child's activities with tables and practical experiences with capacity should be extended to include a consideration of fractional forms.

8 pints = 1 gallon
4 pints = $\square$ gallon
2 pints = $\square$ gallon
1 pint = $\square$ gallon

---

pints = $\frac{1}{2}$ gallon
pints = $\frac{1}{4}$ gallon
pints = $\frac{3}{8}$ gallon

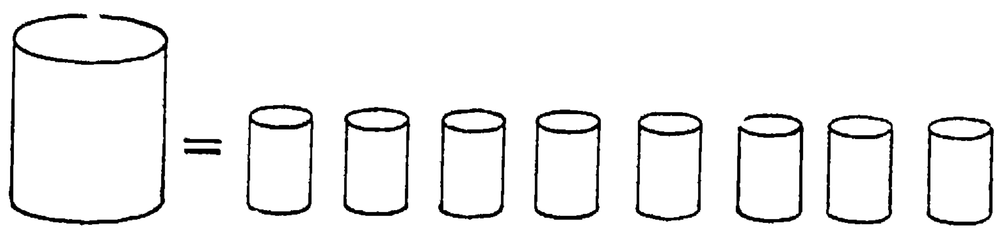
$1\frac{1}{8}$ gallons = $\square$ pints = $\frac{\square}{8}$ gallon

$1\frac{1}{2}$ gallons = $\square$ pints = $\frac{\square}{8}$ gallon

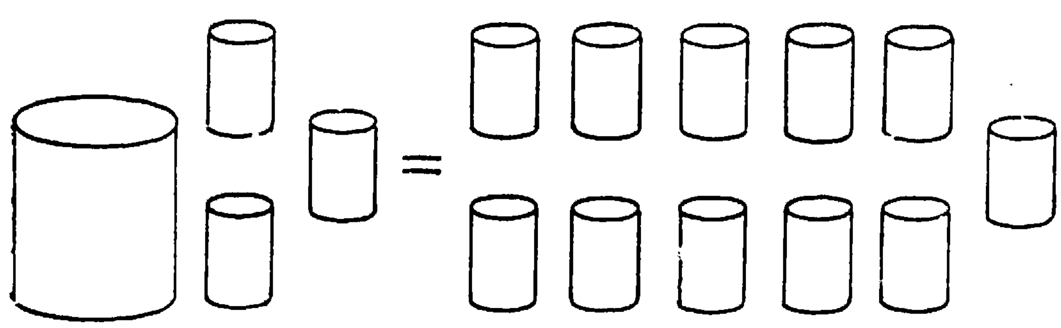
This aspect of the work should be linked with the activities undertaken with fractions in pure number.

3. Reduction

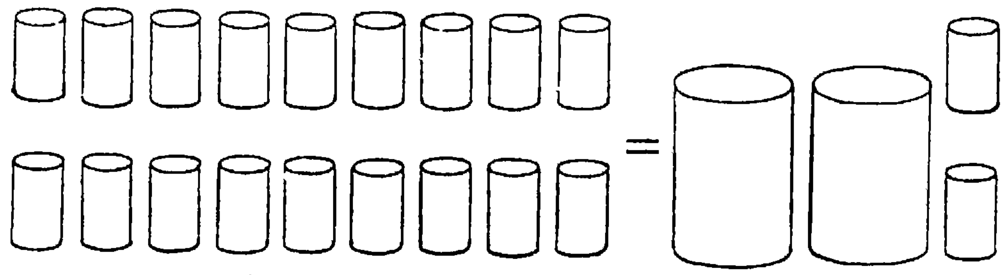
(a) In this section the child's first experiences with reduction should be limited to practical situations.



1 gallon = 8 one pints



1 gallon 3 pints = $\square$ pints



18 pints = $\square$ gallons $\square$ pints

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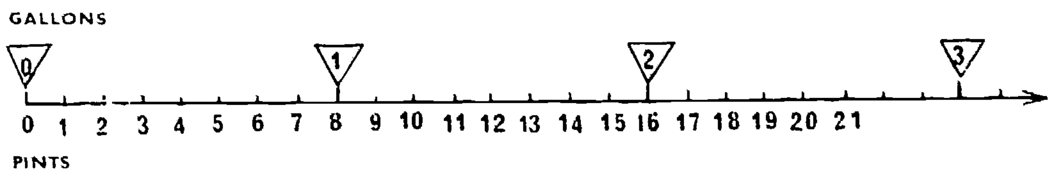
(*b*) Later, labelled number lines might be used to assist the child in visualizing related measures in a particular situation.



$$2 \text{ gallons } 2 \text{ pints} = \square \text{ pints}$$
$$27 \text{ pints} = \square \text{ gallons } \square \text{ pints}$$
$$2\tfrac{1}{2} \text{ gallons} = \square \text{ pints}$$

(*c*) Finally the child might be asked to record on paper how he obtained his answer.

When asked to solve the following examples he may, or may not, use and record the steps given below. He should be encouraged to find various mathematical ways of obtaining the correct answer.

$$\begin{aligned} 5 \text{ gallons} &= \square \text{ pints} \\ 5 \text{ gallons} &= 5 \times 8 \text{ pints} \\ &= 40 \text{ pints} \end{aligned}$$

$$\begin{aligned} 3\tfrac{1}{4} \text{ gallons} &= \square \text{ pints} \\ 3\tfrac{1}{4} \text{ gallons} &= (3\tfrac{1}{4} \times 8) \text{ pints} \\ &= (3 + \tfrac{1}{4}) \times 8 \text{ pints} \\ &= 3 \times 8 + \tfrac{1}{4} \times 8 \text{ pints} \\ &= 24 + 2 \text{ pints} \\ &= 26 \text{ pints} \end{aligned}$$

$$\begin{aligned} 3 \text{ gallons } 3 \text{ pints} &= \square \text{ pints} \\ 3 \text{ gallons } 3 \text{ pints} &= 3 \times 8 \text{ pints} + 3 \text{ pints} \\ &= 24 \text{ pints} + 3 \text{ pints} \\ &= 27 \text{ pints} \end{aligned}$$

$$\begin{aligned} 29 \text{ pints} &= \square \text{ gallons} \\ 29 \text{ pints} &= \tfrac{29}{8} \text{ gallons} \\ &= \tfrac{24+5}{8} \text{ gallons} \\ &= (\tfrac{24}{8} + \tfrac{5}{8}) \text{ gallons} \\ &= (3 + \tfrac{5}{8}) \text{ gallons} \\ &= 3\tfrac{5}{8} \text{ gallons} \end{aligned}$$

$$\begin{aligned} \text{Alternatively, } 29 \text{ pints} &= (24 + 5) \text{ pints} \\ &= (3 \times 8 + 5) \text{ pints} \\ &= 3 \text{ gallons} + \tfrac{5}{8} \text{ gallon} \\ &= 3\tfrac{5}{8} \text{ gallons} \end{aligned}$$

WEIGHT

AIMS

1. To extend the child's personal experience of estimating, comparing, and weighing objects. Such experience will include the introduction of the stone and also the ton (ton—vocabulary usage only).

2. To develop ability to carry out one-step reductions, with the incidental use of fractional forms.

NOTES

1. Ability to estimate a weight with a fair measure of accuracy is important in itself. It also provides a useful rough check on the accuracy of later weighing.

2. Computational work should grow out of personal experience in estimating and weighing and should not be considered merely as an exercise in calculation.

3. Simple problems involving fractional forms, one-step reduction, or practical experiences form part of the course.

DEVELOPMENT AND PRACTICAL EXPERIENCE

1. Experiences in Estimating and Measuring

(a) Before a child actually weighs objects he should be encouraged to make estimates of their weights; the process of weighing acts as a check upon the estimate made. Once the check is made attention should be directed to the size of the error. Effort should be made to reduce errors in estimation by frequent practice. The construction and completion of a table such as that which follows will be of assistance.

Object	*Estimated Weight*	*Actual Weight*	*Error in Estimate*
Brick ..	10 lb.	7 lb.	3 lb.

(b) A variety of measuring instruments should be available and used by all children, for example, kitchen and bathroom scales. One of the measuring instruments should be a balance.

(c) Some children might undertake the construction of simple weighing machines, using scrap material. Two simple examples are presented on page 91 as a guide to what could be done. Children will, in fact, advance suggestions of their own.

By constructing a weighing machine, the child may solve problems that reflect many of the basic ideas involved in measurement. Through solving these problems it is likely that the child's understanding of the topic will be enhanced. For example :

How will the instrument be calibrated ?

Why is a particular unit of measure adopted ?

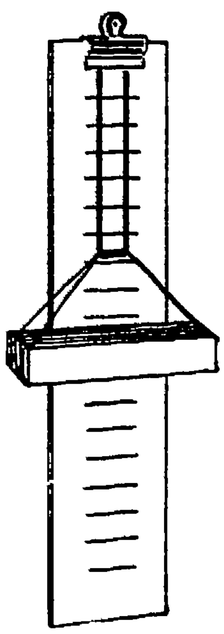
Has this unit of measure any advantage over the various units adopted by other people who build weighing machines ?

Is the same measure given to the same object when weighed on a neighbour's scale ?

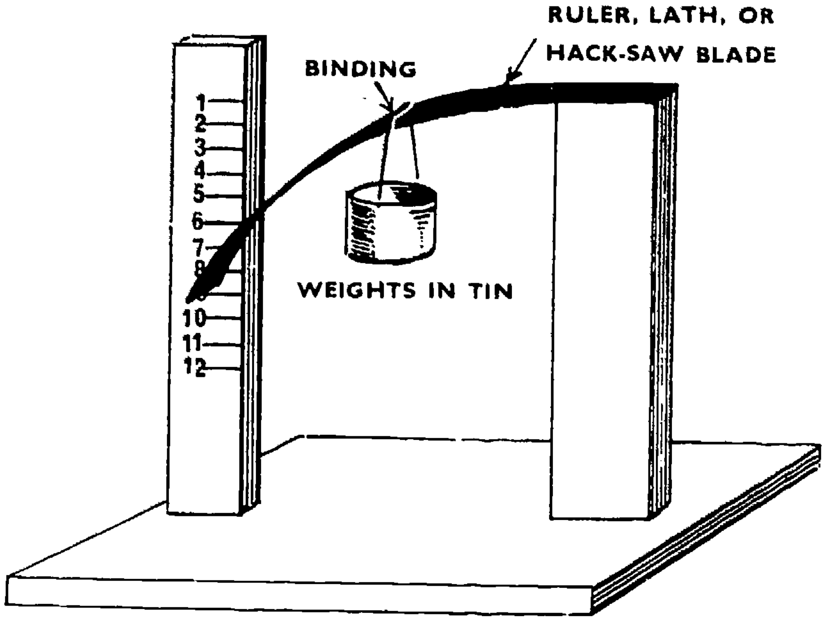
(i) The "Elastic Band" Machine

Material used—

yard ruler, bulldog clip, elastic band, chalk, matchbox, and string.



(ii) Hack-saw Blade Weighing Machine



Other interesting questions that arise out of building a weighing machine are correlated with other subject areas. For example :

How can this simple machine be used to make measures of small magnitude ?

What would happen to the measure given to an object if the position of the weighing pan on the ruler was altered ?

If an object is repeatedly measured over a series of days with the same instrument, is there any change in the measure ? If there is change, how can this change be explained ?

(*d*) Other practical weighing situations might be to find—

(i) the weight of a gallon of water ;

(ii) the weight of ice-cubes compared with the weight of the water used to form the cubes,

(iii) the weight of a house brick dry and the same brick after soaking it in water,

(iv) the weight of two dollars' worth of twenty-cent coins.

(e) To provide a more challenging problem, limits can be placed upon the actual weights available with the balance used, for example, using only an ounce weight and paper-clips to find the weight of one paper-clip.

2. Introduction of New Units

14 lb. = 1 stone

ton—vocabulary usage only.

(*a*) In introducing the child to the new unit of measure—the stone—use may be made of his knowledge of the pound.

Fourteen one-pound bags of sand could be placed in a bucket. The child then lifts the bucket to gain some appreciation of the weight, which is given the name of "one stone".

Some children may wish to discuss why the weight being lifted is only an approximation to one stone. Ways of overcoming the inclusion of the bucket weight could be considered.

(*b*) The child might then be encouraged to find common objects that weigh approximately one stone. A group list or a labelled collection of these objects could be made.

(*c*) The weight of objects greater than one stone might then be considered. A child may estimate the weight of his friend, and then check this against the known or measured weight.

A link could be made at this point with graphs. Children could be encouraged to make a graph of the estimated weights of six boys or six girls and a graph of the true weights of the same people.

(*d*) To introduce the ton, children might consider the weight of cars. Smaller cars such as the Mini-minor and the Volkswagen weigh a little more than half a ton, while cars such as the Holden and the Falcon weigh more than one ton. Twenty bags of briquettes weigh a ton. Twenty-four bags of cement weigh about one ton. The weights of loads carried by trucks might also be considered.

3. Reductions

Reduction should be developed in a practical way concerned directly with weighing. It should be thought of as renaming the weight measure in terms of a particular unit (for example, the ounce) or as renaming, using as few weights as possible.

Experiences relating weight to space occupied should continue, using such materials as sand, wheat, bran, lead shot, and water.

While the child should be encouraged to record his arithmetical procedures, too much emphasis should not be placed on the written record at this stage. Written work should be within the range of the child's knowledge of tables and might be set out in equation form as presented below.

$$\begin{aligned} 5 \text{ stone } 8 \text{ lb.} &= 5 \text{ stone} + 8 \text{ lb.} \\ &= (5 \times 14) \text{ lb.} + 8 \text{ lb.} \\ &= (10 \times 7) \text{ lb.} + 8 \text{ lb.} \\ &= 70 \text{ lb.} + 8 \text{ lb.} \\ &= 78 \text{ lb.} \end{aligned}$$

TIME

AIMS

1. To extend the child's experience of, and improve his ability in, estimating various passages of time.

2. To develop—

(a) knowledge of the tables set out below ;

(b) ability to carry out one-step reduction within the limits of the tables ;

(c) an understanding of the importance of the calendar in practical activities of the human race.

DEVELOPMENT

1. Practical Experiences

(a) It is important that the child's skill in estimating the duration of time is maintained and improved. He might be asked—

" How long will it take to write your name ten times ? " or

" How long will it take for you to walk from the classroom to the flag-pole and back again ? "

Once the child has made his estimate he should be encouraged to time the activity, using an appropriate instrument.

(*b*) A variety of working measuring instruments should be used by all children. Instruments such as egg-timers, other cooking timers, clocks, and stop-watches should be used if available. By using such devices, points such as the following can be emphasized :

(i) the duration of time ;

(ii) the suitability of various instruments for particular purposes ;

(iii) the need for a common unit of measure ;

(iv) the greater precision that can be obtained by using a smaller unit of measure.

One example of this type of activity is as follows. Children investigate the time a balloon takes to reach the ground after being released from the hand of a boy standing on a chair. To measure this the children are asked to use "number of claps", an egg-timer, a clock, and a stop-watch. These "devices" are to be used in the order given.

From this activity the children should find that claps are unsuitable as units for measuring the duration of time. Different children may label the same event with a different number of claps. If the experiment is repeated a number of times and the rate of the claps is varied, the unsuitability of the measuring unit is again emphasized.

When the egg-timer and the clock are used, the child discovers that the unit of measure may be too large. It is when the stop-watch is used that the child has a measure that is sufficiently precise.

(*c*) Just as children improvised machines to measure weight, so they can undertake the construction of machines to measure time. In making these instruments the child should consider problems such as the following :

(i) Is the design of the instrument suitable for a unit of measure ? For example, it is impossible to graduate a jam jar into units based upon the amount of water that drips into the jar in four seconds. A modification might be to improve the construction of the water-clock by replacing the jar with a narrow glass tube so that use of smaller units is possible. By refining the measuring instrument we can improve the measurement of time.

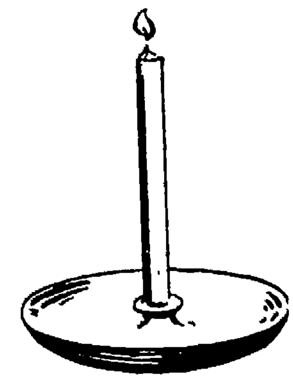
(ii) Are there advantages in having a unit common to all machines ? If there is no such unit how can comparisons be made between the measure read from one instrument and that recorded on another instrument ?

(iii) Should a standard unit of measure be used, such as a minute or a second ? If so, how will the instruments made be calibrated ?

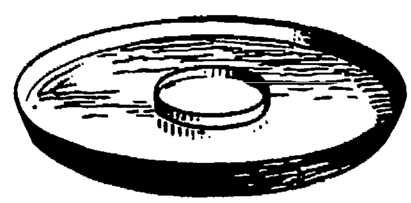
(iv) What is the nature of the quantity being measured ?

Three examples of measuring devices that children might make are as follows :

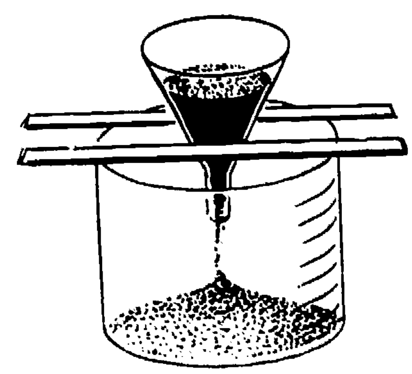
(i) Candle clock—A candle may be placed in a tin lid with the units of measure being indicated by pins pressed into the wax of the candle



(ii) Water-clock — A water-clock can be made from a bowl and a tin lid. A small hole is pierced in the base of the lid and the lid is then floated in a bowl of water. The unit of measure is the time taken for the lid to sink to the bottom of the bowl.



(iii) Sand clock—A sand clock can be made from a funnel, a jar (of glass or clear plastic), two rulers, and a given quantity of sand. The rulers are placed across the mouth of the jar and the funnel supported between them. Graduation marks are placed on the outside of the jar with a marking pen or an oil pastel.



(*d*) An extension of these practical experiences may lead the child to consider questions such as—

A thousand seconds—how many hours ?

A thousand minutes—how many days ?

A thousand days—how many months?

2. Skills

The child should be given practice in telling the time correct to the nearest minute, in reading a calendar, and in recording the date. A large calendar for the entire twelve months should be a feature of every room. This will enable the child to see the names of all the months, the order in which they come, and the number of days in each. Many dates should be located on the calendar and associated with the days of the week. The child should discover that there are three hundred and sixty-five days in a year (three hundred and sixty-six days in a leap-year). Some children may wish to find out why there is need for a leap-year.

The calendar may be used to show the children's birthdays and events of local and of national importance. Questions such as "How many weeks (or days) until Christmas ?" are useful.

3. Reductions

Knowledge of the following tables is expected in Section G :

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
7 days = 1 week
14 days = 1 fortnight
52 weeks and 1 day = 1 year
52 weeks and 2 days = 1 leap-year
Number of days in each month.

Reductions may be thought of as renaming the time measured, using different units of measure. Initially, examples may be solved by reference to some measuring instrument. Questions such as the following might be asked :

How many seconds in three minutes ?

How many days in five weeks ?

How many weeks and days in 37 days ?

One hundred and sixty seconds ; how many minutes and seconds ?

How many minutes in two and one-quarter hours ?

Some children may wish to record the procedure they used to find the answer to a question. However, too much stress should not be placed upon the written record at this stage.

$$\begin{aligned} 2\tfrac{1}{4} \text{ hours} &= 2\tfrac{1}{4} \times 60 \text{ minutes} \\ &= (2 \times 60 + \tfrac{1}{4} \times 60) \text{ minutes} \\ &= (120 + 15) \text{ minutes} \\ &= 135 \text{ minutes.} \end{aligned}$$

4. Enrichment

A few children may wish to pursue further the topic "Time" and investigate questions of their own or questions similar to the following :

(a) When was the first mechanical clock made ?

(b) How did the small country communities in Victoria keep the time before 1860 ?

What effect did the coming of the railways have upon this ?

(c) How might a simple alarm clock be constructed from scrap materials ?

(d) What role does time play in sport ?

Can the various sports do without timing devices ?

What units of time do the various sports use ?

(e) How does a clock work ? (Children may investigate the mechanism of a clock to solve this problem.)

MONEY

AIM

To continue the study of money, with emphasis on relationships and equivalents, and to introduce the formal processes in money.

COMMENTS

1. By the time children have reached Section G they should—

(*a*) be able to recognize and name the various coins and the dollar note ;

(*b*) know the relatives values of the coins and the one-dollar note ;

(*c*) be able to count money to one dollar ;

(*d*) have mastery of money notation ;

(*e*) be able to carry out simple money computations involving addition, subtraction, and multiplication.

In Section G the child's knowledge and skill are extended to include the ability—

(*a*) to recognize and name all notes ;

(*b*) to carry out money operations involving the four processes, the amounts being limited to $10·00 with multipliers and divisors to 10 ;

(*c*) to solve problems involving money.

2. At this stage, processes should be carried out in dollars and cents and not as decimal operations.

3. The decimal point should appear in all computations involving dollars and cents. Children should be encouraged to align decimal points as in the following examples :

$ c	$ c	$ c
3·21	5·73	1·25
·46	·14	3
1·21	——	——
——	5·59	3·75
4·88	——	——
——		

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DEVELOPMENT

1. Writing Amounts in Dollars and Cents

(a) In Figures

$59 or $59·00

$0·26 or 26 cents or 26c

$0·08 or 8 cents or 8c

$0·60 or 60 cents or 60c

The use of zero as a place holder should be stressed. In decimal currency where dollars and cents are involved there should always be two digits after the decimal point. The cent point in money will generally be recorded as for decimal fractions, e.g., $6·52. In banking transactions, for example, in completing withdrawal forms in school bank accounts, the dash (—) is preferred to the decimal point, $6—52. In printed material such as newspapers the cent point will usually be recorded on the line and children should be familiar with this notation, e.g., $6.52.

(b) In Words

(i) Three dollars thirty-four cents ;

(ii) five dollars three cents.

Examples such as the following may be given to children :

(i) Write nine cents in figures in two different ways.

(ii) Using the symbol $, write down these amounts :

three dollars sixty cents
5 dollars 15 cents
four dollars 07
eight cents
17 cents
40c

2. Money Relationships :

(a) Dollar-cent Relationship

e.g. $1·00 = 100 cents
$4·63 = 463 cents
300c = $3·00
296c = $2·96

(b) In Terms of Various Coins

e.g. $1·00 = □ cent coins
$1·00 = □ ten-cent coins
$2·35 = □ fifty-cent coins + □ ten-cent coins + □ five-cent coins
□ = 1 two-dollar note + 1 ten-cent coin + 3 two-cent coins + 1 one-cent coin

(c) Fractional Parts

1 cent / $0·01 } $= \frac{1}{100}$ dollar

10 cents / $0·10 } $= \frac{1}{10}$ dollar

7 cents / $0·07 } $= \frac{7}{100}$ dollar

25 cents / $0·25 } $= \frac{25}{100}$ dollar $= \frac{1}{4}$ dollar

231 cents / $2·31 } $= \frac{231}{100}$ dollar $= 2\frac{31}{100}$ dollars

3. Processes

With the majority of children, processes in money will follow on easily from processes involving whole numbers. As it is important to encourage a variety of approaches rather than restrict children to one particular method, it is desirable to allow children to use the free techniques acquired in the solution of equations (see Section F, Equations) as steps leading to the formal processes in money.

Children will recognize and write amounts in decimal currency notation, but they should carry out processes as for dollars and cents, not necessarily as decimal operations at this stage. All examples should be suitably graded as for whole numbers.

(a) Addition

(i) $3·25 + $4·95 = $3 + 25c + $4 + 95c
= $3 + $4 + 25c + 95c
= $7 + 120c
= $7 + $1 + 20c
= $8 + 20c
= $8·20

(ii)		(iii)
$3·25	$3 + 25c	$ c
$4·95	$4 + 95c	3·25
	———	4·95
	$7 + 120c	——
	$7 + $1 + 20c	8·20
	$8·20	——

(iv) $\begin{array}{r} \$ \quad c \\ 3{\cdot}25 \\ 4{\cdot}95 \\ \hline \\ \hline \end{array}$

Think of \$3·25 as 325 cents and \$4·95 as 495 cents and add as for any two whole numbers. Then rename using the dollar sign.

Understanding of the dollar-cent relationship, as given in 2 (*a*) above, is an essential prerequisite for this method.

(*b*) *Subtraction*

Note that the child may use more than one method in solving computational examples. In fact, he should be encouraged to vary the method used according to the nature of the example. However, when a general method is required, and the need for this becomes greater as the examples increase in complexity, equal additions is the method favoured. See Formal Processes, page 39, for a discussion of the "equal additions" method.

$$\begin{array}{r} \$ \quad c \\ 4\cdot\ 5^{1}3 \\ 1\cdot{}_{1}2\ 7 \\ \hline 3\cdot\ 2\ 6 \\ \hline \end{array}$$

The sequence of development used to reach the end point given above would be similar to that given in addition.

The following examples indicate some of the variations that children may use.

(i) $\$4{\cdot}35 - 85c = \square$

$$\begin{aligned} \$4{\cdot}35 - 85c &= (\$4{\cdot}35 + 15c) - (85c + 15c) \\ &= \$4{\cdot}50 - \$1{\cdot}00 \\ &= \$3{\cdot}50 \end{aligned}$$

Here the child has made the subtrahend equal to one dollar by adding fifteen cents. To maintain the original difference between the minuend and the subtrahend, fifteen cents has been added to the minuend. This then utilizes a property of differences, namely, that if equals are added to unequals the original difference remains unchanged.

(ii) $\$2{\cdot}13 - 67c = \square$

In this example a child may adopt a complementary additions approach. "What must I add to 67c to equal two dollars thirteen cents ?"

$$\begin{aligned} \$2{\cdot}13 - 67c &= 33c + \$1 + 13c \\ &= \$1{\cdot}46 \end{aligned}$$

(iii) $\$7{\cdot}00 - \$1{\cdot}36 = \square$

A common approach to such an example is to calculate the dollars first and then rename one dollar in cents from which the remaining thirty-six cents is subtracted. Note that the steps given below would not necessarily be recorded by a child. They are given so that the sequence of thought is made clearer.

$$\begin{aligned}\$7{\cdot}00 - \$1{\cdot}36 &= (\$7{\cdot}00 - \$1{\cdot}00) - 36c\\ &= \$6{\cdot}00 - 36c\\ &= (\$5{\cdot}00 + 100c) - 36c\\ &= \$5{\cdot}00 + (100c - 36c)\\ &= \$5{\cdot}00 + 64c\\ &= \$5{\cdot}64\end{aligned}$$

(*c*) *Multiplication*

(i) $$\begin{aligned}74c \times 6 &= (70c + 4c) \times 6\\ &= (70 \times 6)c + (4 \times 6)c\\ &= 420c + 24c\\ &= 444c\\ &= 400c + 44c\\ &= \$4 \quad + 44c\\ &= \$4{\cdot}44\end{aligned}$$

(ii)

$0·74	
x 6	
24c	6 x 4c
420c	6 x 70c
444c	6 x 74c
$4·44	

(iii)

$$\begin{array}{r}\$0{\cdot}74\\ \times\ 6\\ \hline \$4{\cdot}44\\ \hline\end{array}$$

(*d*) *Division*

(i) Simple oral exercises within the number experience of the children.

(ii) Simple recorded exercises involving renaming.

e.g., $$\begin{aligned}\$4{\cdot}08 \div 4 &= (\$4 + 8c) \div 4\\ &= \$(4 \div 4) + (8 \div 4)c\\ &= \$1 + 2c\\ &= \$1{\cdot}02\end{aligned}$$

(iii) When division of whole numbers can be handled, children should be able to proceed to examples such as the following :

$3·54 ÷ 6

Rename $3·54 as 354 cents and divide

Answer—59c or $0·59

$$\begin{array}{r|l} 6\ \overline{)354} & \\ 300 & 50\\ \hline 54 & \\ 54 & 9\\ \hline & 59\end{array}$$

4. Problems

Shopping activities should include both oral and recorded work, and at this stage should provide a basis for process work in decimal currency.

Shopping activities should include listing articles (to three items) and totalling the prices. Usually only one operation should be used.

Problems could include examples such as the following :

(*a*) Tom plays in the school football team and needs new boots. Those he likes cost \$7·45. Tom has saved \$9·00. How much money will he have left if he buys the boots ?

(*b*) Use the information on the two dockets given below to answer each of the following questions.

Mrs. Maley	
4 lb. peas ..	52c
2 lb. apples ..	36c
2 bunches carrots	30c

Mrs. Mancini	
3 lb. beans ..	60c
5 lb. potatoes	40c
1 lb. bananas	12c

(i) How much greater is the cost of Mrs. Maley's order than Mrs. Mancini's ?

(ii) If Mrs. Mancini increases her order to include 5 lb. of beans and not three pounds, how would the total cost of her order compare with Mrs. Maley's ?

(iii) How much per lb. did Mrs. Maley pay for her peas ?

(iv) If Mrs. Maley ordered 9 lb. of apples for next week at the current price, what would she have to pay ?

SPATIAL RELATIONS

AIMS

1. To develop the child's awareness of space and his appreciation of shape in his environment.

2. To further familiarize the child with his environment so that he can confidently supply accurate directions to places he knows and produce simple sketch-maps illustrating these directions.

CONTENT AND DEVELOPMENT

A. Techniques for, and Practice in, the Accurate Ruling and Measurement of Lines

(*a*) Straight lines should be ruled, accurate to $\frac{1}{10}$ inch.

(*b*) Ruled lines and measurement should not be confined to horizontal lines.

(*c*) Techniques for measuring curved lines should be investigated.

Reference should be made to Length, Sections E, F, and G.

B. Some Properties of Common Shapes

In this context "shape" refers to both three-dimensional and two-dimensional forms. Through Sections A to F the child has observed objects in his environment and has learned to distinguish common elements of shape in them. In Section G these observations should continue and more precise language should be used to describe such observations. Statements such as the following would be acceptable :

(*a*) This book is of rectangular shape.

(*b*) Some rectangles are squares.

(*c*) Squares have four sides of equal length.

(*d*) A cube has twelve edges of equal length.

(*e*) A sphere has no edges.

(*f*) These tiles (regular hexagons) fit together to cover a surface completely.

(*g*) Pencils, drinking straws, jam tins, and coins have the shape of a cylinder.

(*h*) A container having triangular faces can be constructed from a cylinder by suitable creasing (see some milk containers).

Responses such as these should arise from planned experiences. However, the child should be given much opportunity for free experimentation with materials to build up the rich experience necessary to facilitate understanding of more formal studies at later stages.

Materials used in such directed and informal activities may well include constructional material such as interlocking blocks ; perforated strips with bolts to join them ; jig-saw puzzles based on geometric figures ; wooden, plastic, or card shapes ; wooden or plastic cubes, and similar objects. Informal exploratory activities and more formal activities associated with specific problems may be treated in quite small sessions, say of five to ten minutes' duration, once or twice a week over the year. If interest is aroused, much of the experience can be obtained in extra-curricular activities.

C. The Construction of Shapes Made Up from Triangles and Rectangles

The child should be given the opportunity to experiment with tiles (terra-cotta, wood, card, plastic) of various regular shapes to cover surfaces and to construct patterns. He should become aware that any triangular or rectangular shape can be used to cover a surface completely.

Exercises such as the following will assist in making children familiar with properties of triangles :

A child or a small group of children is given a number of triangular shapes. The collection contains a variety of sizes and shapes. Included however are many of the same size and shape—equilateral, right-angled, isosceles, scalene. Children are challenged to construct :

(*a*) a hexagon,

(*b*) a square,

(*c*) a large triangle,

(*d*) a five-sided figure,

(*e*) a hexagon which has sides of equal length,

(*f*) the figure having the fewest (or greatest) number of sides possible, using six equilateral triangles,

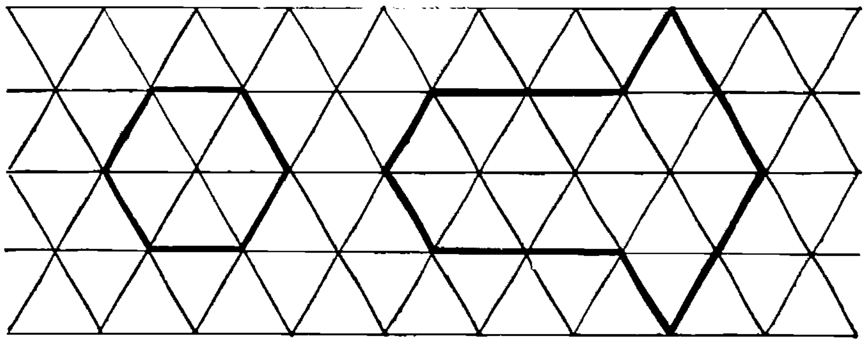
(*g*) the longest straight line that can be made using five triangles,

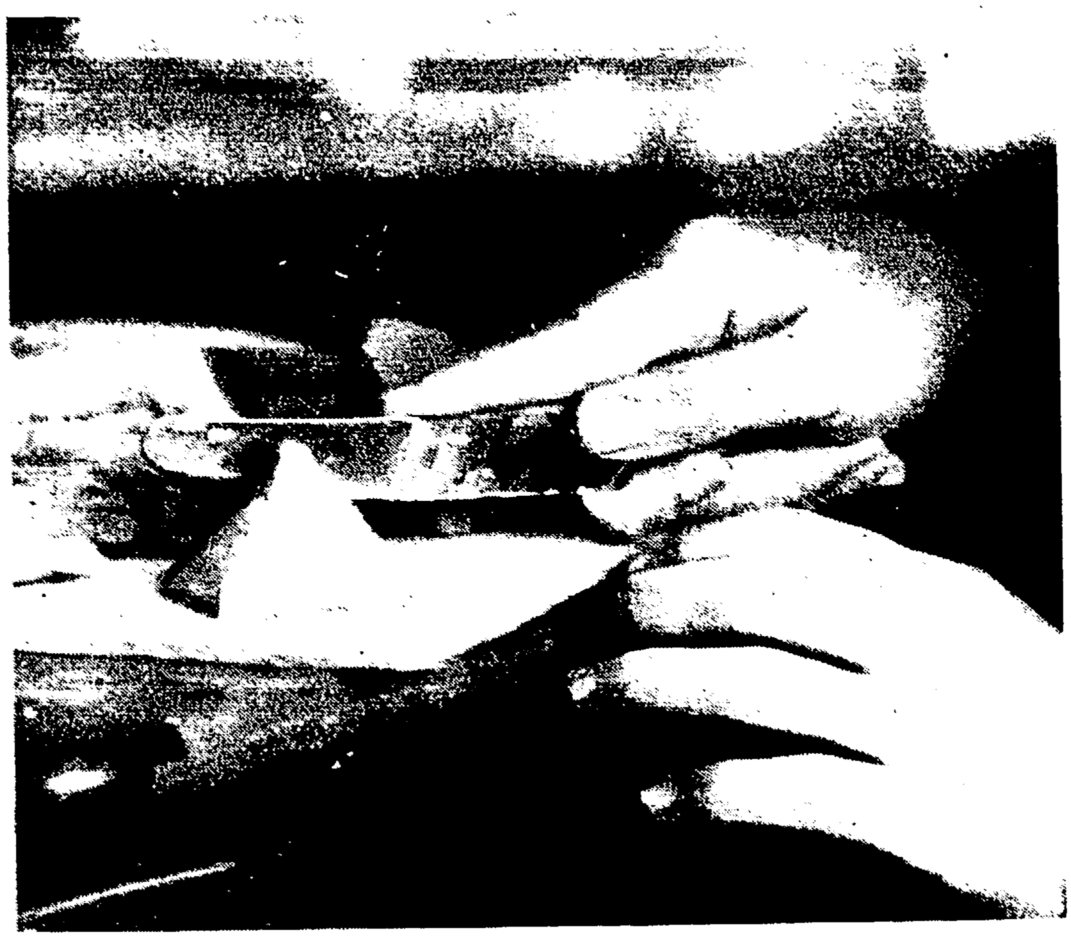
(*h*) the smallest perimeter that can be made using four given triangles,

(*i*) a continuous surface cover using congruent scalene triangles.

D. The Use of Nets

Nets should be used to assist the discoveries that relatively complex patterns and shapes may be composed of smaller basic shapes and that such patterns and shapes may be decomposed in some cases into smaller, simpler units.





In this context a "net" is an over-all pattern of identical shapes such as rectangles or triangles. In this section it will be generally more desirable for the child to use prepared nets rather than to prepare them for himself. The nets may be prepared on paper or on transparent sheets. The child should discover that nets composed of equilateral triangles are suitable for regular hexagonal figures. With the addition of further lines, certain types of right-angled triangles and rectangles can be obtained from them ; similarly, nets made up from rectangles can be used to produce octagonal figures and isosceles, right-angled triangles. The nets can be used also for informal units in the measurement of area and in area/perimeter considerations.

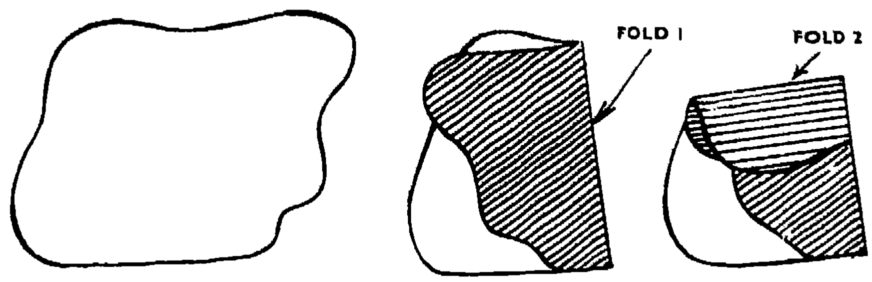
Information will generally be acquired as a result of incidental and informal pattern work. However, patterns prepared by the teacher could be examined by children in an endeavour to isolate the basic shapes used.

E. Measurement of Angles

(Angles and their measurement in terms of turning through full turns, half turns, and quarter turns.)

The basic angle which will concern children most at this level is the right angle. This angle is observed in a great number of environmental situations. A model for comparison purposes can easily be made from a piece of paper by folding the paper once and then folding a second time along the first crease.

Thus



Using this model other angles may be compared and, if desired, measured in units of right angles or even, by folding again, in half right angles. Generally the turning is considered in an "anti-clockwise" direction. The term "revolution" adequately serves as a unit of measure.

Thus, measurements of angles may be :

(i) a right angle or a quarter revolution,

(ii) two right angles or a half revolution,

(iii) four right angles or one revolution.

Activities in which ideas of angles may be developed in this section include direction games (e.g., treasure hunts) in association with north, south, east, and west, and the description of shapes (e.g., a rectangle has four right angles).

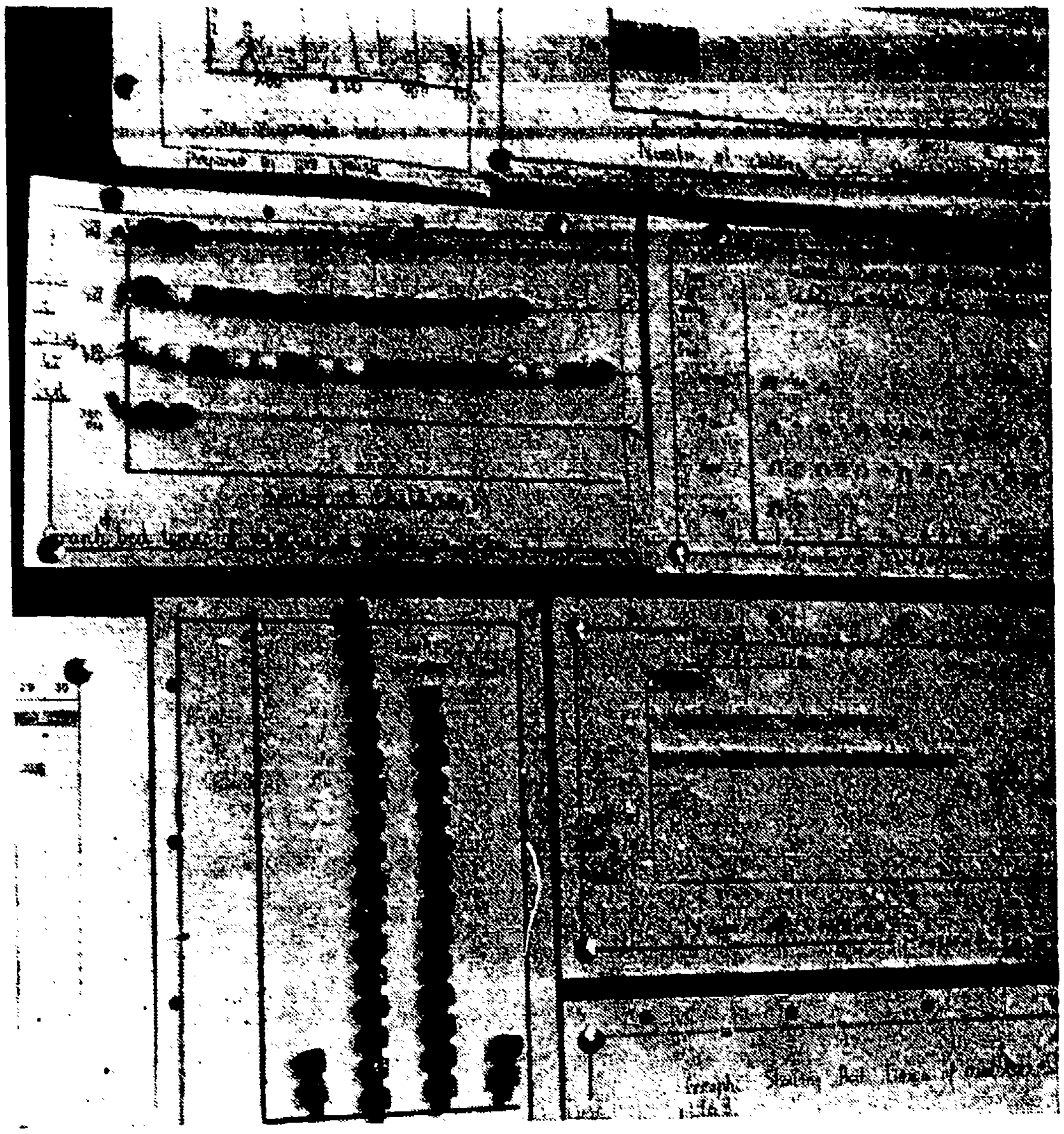
F. Directions

(The ability to give accurate, simple directions to specific locations, to prepare simple maps, and to follow spoken or written directions.)

It is important to begin with the child's immediate environment and with locations within his field of vision. Care should be taken to ensure that directions given (either by a child or by a teacher) are clear and unambiguous. Very simple problems should be considered first. These may take the form of a "treasure hunt" where all directions are written ; or of the description by the child of the location of a point within the classroom. Terms such as left, right, north, south, east, west, between, on, above, and below should be used confidently and accurately.

Where scale is used it should be approximate only. Thus a room plan may be roughly square or oblong as the actual room appears to the child.

Activities should be simple rather than complex and taken regularly for periods of short duration, say for fifteen minutes. However, very many opportunities present themselves for the practice of correct, precise terminology and the teacher should be constantly alert to develop such vocabulary.



STATISTICS AND GRAPHS

AIM

To develop the child's ability to construct and interpret simple bar and pictorial graphs.

NOTES

1. As a result of his experiences in this topic, the child should build up certain understandings about graphs.

(a) Graphs give information in a concise form. Some children may question the need for a graph because the information presented can be easily stated in words or adequately interpreted from statistics. Initially, the child will be aware that the graph is a way of picturing information. Later, with the introduction of more complex graphs, the concise nature of the method of presentation will be more apparent.

(b) Graphs show relationships. By graphing information, relationships that were not previously clear can become obvious. Patterns can be sought and predictions made.

This particular aspect of graphical understanding will arise out of the experience the child has in sorting the information he gathers.

(c) One type of graph may be more appropriate than another in presenting information. At this level the child's understanding of this aspect will be limited. For example, a bar graph is most suitable when changes are not continuous and is best used to display unrelated data such as colour of hair of children in the grade. A pictorial graph often lacks accuracy but presents the information simply and attractively.

2. The material used in this topic will often arise out of the child's experiences in other subject areas. Science and social studies are two subjects that provide opportunities for gathering, organizing, and interpreting information.

3. An essential part of the topic "Statistics and Graphs" is the discussion of the information presented.

Example :

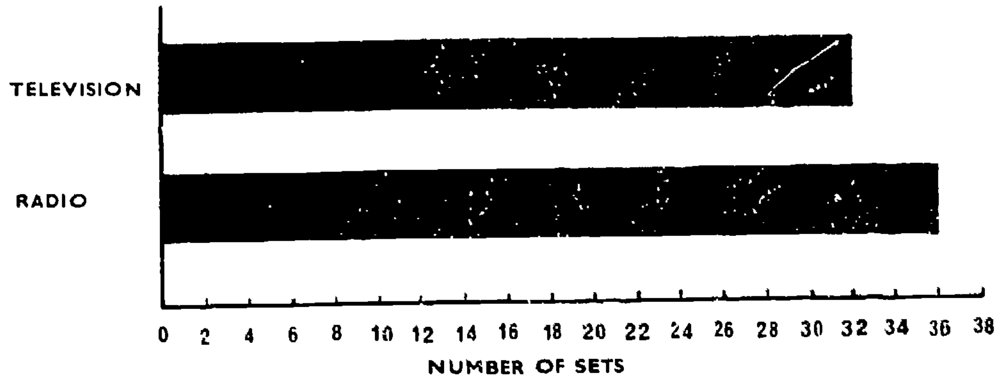
How Grade IV Children Travel to School

How do Grade IV children travel to school ? In what grade are the children ? What means of travel is used by the greatest number of children ? What is the difference in number between the children who walk to school and those who travel by bus ? Do you think this school would be in the country or in the city ? Why ?

4. In this section, children will construct and interpret two types of graphs.

(a) *Bar Graph.* This graph uses the length of a bar or a line to represent quantity. The bars can represent the numbers of things of two or more separate kinds, such as television sets and radio sets. These bars can be arranged across the page or up-and-down the page.

Graph of the Numbers of Television and Radio Sets in Homes of Grade IV Children



(*b*) *Pictorial Graph.* A pictorial symbol is used to represent either a unit quantity or a quantity larger than one. In Section G, construction of such graphs should be concerned with whole symbols. However, interpretation of simple graphs showing part of a symbol should not be avoided.

Graph of Milk Drunk at School in One Week

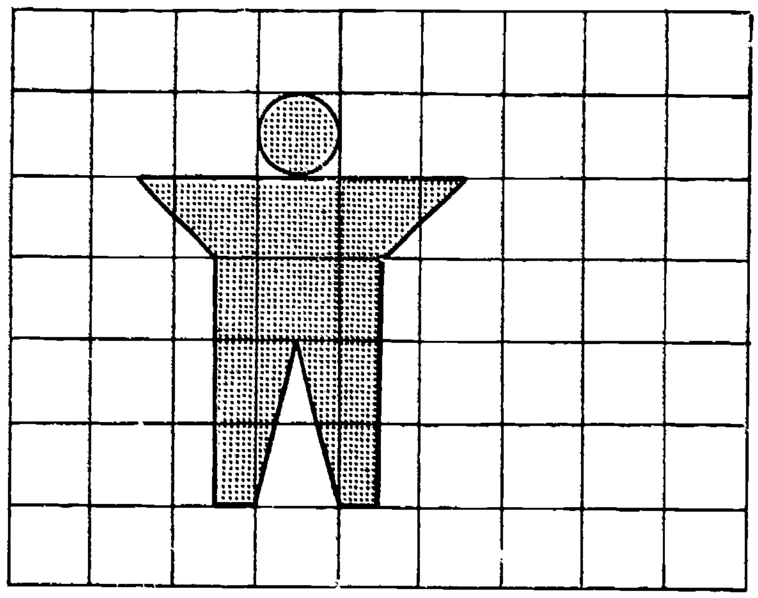
Monday	△ △ △ △ △ △ △ △
Tuesday	△ △ △ △ △ △ △ △ △ △
Wednesday	△ △ △ △ △ △ △
Thursday	△ △ △ △ △ △ △ △
Friday	△ △ △ △ △ △

△ equals 2 one-third pints of milk

5. The making and reading of graphs is a skill that has to be taught and practised. In the choice of activities consideration should be given to the following points :

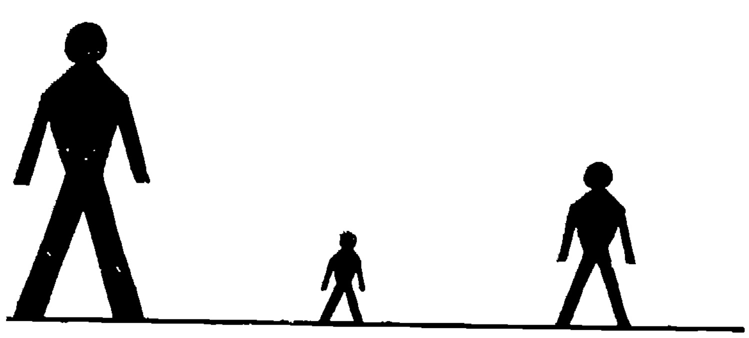
(*a*) Initially, graph paper is unnecessary. At a later stage paper with a network of large squares is suitable. (Graph paper with ten rulings to the inch is not suitable until graphing techniques have been firmly established, possibly late in Section H.)

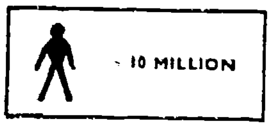
(*b*) Some of the more artistic children should be able to produce their own symbols for use in the construction of pictorial graphs. A simple example built up on squared paper is as follows :



(c) Pictorial graphs using a symbol that indicates a change in quantity by increasing or decreasing its size *should be avoided at this level.*

Populations of Three Countries





(d) Children should be encouraged to adopt the accepted conventions when presenting information in graphical form. This will probably be done best when discussing the actual examples produced by the children. These include matters such as labelling of information, sequence in which the information is presented on the graph, and consistency in the nature of the symbols and the bars used.

DEVELOPMENT

1. A typical activity as an introduction to this topic could be a study of the number of peas in a collection of pods.

Each pupil is given at random three pods from a bag that contains many more pods than those needed. The pupil opens each pod and counts and records the number of peas inside.

Questions such as "Which of your pods has the largest (or the smallest) number of peas in it ?" help to show the variations in the number of peas found.

Ways of recording such information should be discussed with the pupils. Some suggested methods will probably be rather cumbersome.

A number line extending from zero to more than the largest number of peas found in any one pod could be drawn on the chalkboard. Each pupil in turn puts a cross above the numeral that represents the largest number of peas in any of his pods.

Questions such as "Which number of peas appears most frequently ?" should lead the child to realize that this sort of information is readily obtainable from a graph.

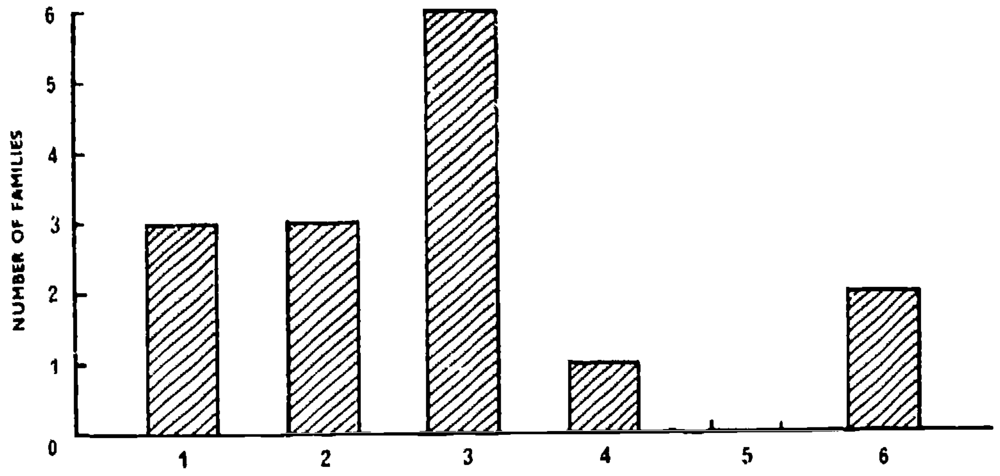
2. Another activity could be the collection and the classification of class information into a table as follows :

Name of Child	*Sex*	*Number of Children in Family*	*Weight (Pounds)*	*Birth (Month)*

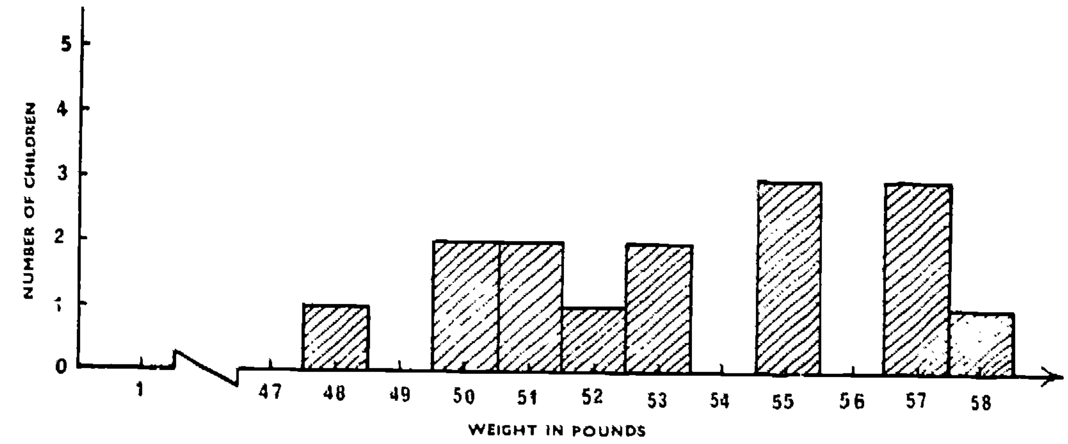
Such data are more easily interpreted when presented in graphical form. From the information gathered, various graphs could be constructed and then interpreted.

With the early work, the information may be written on the chalkboard and the graph built up as a group activity. Examples of typical graphs based on the group information are given below.

Graph of Variation in Family Size



Graph of Variation in Weight



Note that the attention of the child will have to be drawn to the complication in the second graph. Here the axis labelled "weight in pounds" is broken. The child may graph this information using a continuous scale ranging from 0 to 59. When the graph is completed the child should be encouraged to discuss the unbalanced nature of his presentation and to offer suggestions as to how it might be corrected. If necessary, the teacher may introduce the convention used to indicate that a section of the scale has been omitted.

TOPIC SUGGESTIONS

Shoe sizes
Attendances
Arm spans
Milk drinkers
Heights of children
Differences in heights from February to December
Weights of children
Traffic counts (pedestrians, cars, delivery vans, trucks, etc.)
Shadow lengths
Time spent by pupils in travelling to school
Time spent on a number of specified subjects in a week
Temperature at mid-day on ten consecutive days
Heights of seven mountains
Lengths of six rivers in Australia

A. C. Brooks, Government Printer, Melbourne.